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DEVELOPMENT AND APPLICATION OF A COMPUTATIONAL SHEAR BAND MODEL

Prepared by

SRI International
333 Ravenswood Avenue
Menlo Park, CA 94025

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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A computational shear band model has been developed and used to predict the fragmentation response of explosively-loaded cylinders and munitions. The model is based on the physical processes occurring in the material that lead to fragmentation, as observed in experiments in which the fragmentation process was stopped at various stages of development. Good agreement was obtained between measured and computed fragment size distributions for two widely different steels, which suggests that the model can be used to predict

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20.   (continued)

the effects of variation of geometry, strain rate, and internal pressure on the fragmentation yield of munitions.

In addition, fragments of long-rod penetrators of steel and a tungsten-7Ni-3Fe alloy were examined metallographically to establish the failure modes. The observations suggest that shear banding is a principal mode of penetrator failure. Computational simulations of a plate impacting on edge another plate at various oblique angles were performed using the two-dimensional wave propagation code TROTT with the BFRACT fracture model. The results showed the distribution of shear, compression, and bending stresses in the projectile and indicated the regions where tensile fracture or shear banding might occur.

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ACKNOWLEDGMENTS

I SUMMARY

A computational shear band model has been developed and used to predict the fragmentation response of explosively-loaded cylinders and munitions. To provide a basis for the model, an experimental technique was developed for arresting the dynamic expansion of exploding cylinders at desired loads (see Figure 1). The technique was used to infer the phenomenology and to obtain quantitative data for shear instabilities. Shear instabilities were found to nucleate and to grow in HF-1 steel at equivalent plastic strains of about 15% and 30%, respectively. Shear instability surfaces are roughly semicircular, and the surface displacement is proportional to the depth of the shear band surface (see Figure 14).

The computational model for shear instabilities was used in conjunction with a two-dimensional wave propagation code to compute fragment size distributions from exploding cylinders. The model produces fragments by accounting for the nucleation, growth, and coalescence of shear bands according to mathematical expressions inferred from experimental observations and data. Shear surfaces form anisotropically in the deforming material according to the three-dimensional strain state; moreover, as the shear surfaces form, they relax the stresses. The resulting fragment size distributions (Mott plots) are obtained by summing the contributions from all the computational cells.

Four fragmenting cylinder experiments and one fragmenting projectile experiment differing in material, geometry, and explosive, were simulated computationally (see Figures 16 and 17). Good agreement was obtained between computed and observed fragment size distributions (see Figures 18, 20, and 21). These results indicate the basic correctness of the model and suggest that the model can account for effects of variation of geometry, strain rate, and pressure. Further model development should

focus on early stages of shear band nucleation and growth, and on the stress-strain relations of shear-banded material.

Metallographic examination of fragments from HF-1 and tungsten-nickel-iron long-rod penetrators showed that shear banding was a principal mode of projectile failure under oblique and normal impact conditions (see Figures 22 and 27). Computational simulations with the two-dimensional wave propagation code TROTT were made of a plate impacting on edge a second plate at angles of 30°, 45°, and 55°. Both high tensile and high shear stresses are produced near the impacting edge of the projectile plate, indicating potential fracture and shear banding regions. These stresses are significantly larger if no slippage occurs between the impacting surfaces. Future computations should be performed with shear banding and fracture subroutines and the results should be compared with metallographic observations of impacted projectiles.

II INTRODUCTION AND OBJECTIVES

Shear banding, the development of internal regions of intense plastic strain, is a prominent mechanism of deformation and fracture leading to the disintegration of exploding rounds, long-rod penetrators, and armor plates.[1] The Ballistics Research Laboratory, the Navy Surface Weapons Center, and the Advanced Research Projects Agency are therefore seeking to acquire an understanding of the phenomenon, and have contracted SRI to develop a computational model that can be used to predict the failure behavior of ordnance structures and materials by this mechanism. The model could be used to predict results of given projectile/target encounters and the effectiveness of fragmenting rounds in given scenarios, thus allowing more accurate vulnerability and lethality assessments. Furthermore, a computational shear band model would be useful in the design of improved weapons and armors and in the development of improved ordnance materials.

SRI efforts to construct a shear band model began in 1974 under contract with the Naval Surface Weapons Center.[2] The resulting model, called SNAG for Shear Nucleation and Growth, treated the competing work-hardening and thermal-softening effects of plastically deforming material, and included plastic threshold strain, shear band velocity, and critical internal energy as parameters. Reasonable predictions of the fragmentation behavior of cylinders of HF-1 steel and Armco iron were realized. The goal of the research program described here was to refine this preliminary model and apply it to compute the fragmentation behavior of exploding rounds and long-rod penetrators.

Our approach was to:

● Conceive and perform experiments in which shear bands were produced and arrested at various stages of their development.

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- Observe shear band phenomenology and make quantitative measurements of such properties as shear band numbers, sizes, and spacings.
- Develop a computational model incorporating mathematical expressions describing shear band behavior.
- Incorporate the shear band model in wave propagation codes.
- Test the model by computationally simulating experiments and comparing computed and measured results.

The following section describes the interrupted exploding cylinder experiments and presents the resulting data on shear band behavior. Section IV describes development of a computional model based on the observations and data from Section III. Metallographic evidence for shear band activity in long-rod penetrators is presented in Section V, and Section VI presents two-dimensional slab-slab impact calculations indicating the distribution of shear, bending, and tensile stresses expected in obliquely impacting rods.

III SHEAR BAND MODEL DEVELOPMENT - EXPERIMENTAL PHASE

Our goal for this phase of the program was to devise experimental techniques that would allow us to study the shear banding phenomena from nucleation through growth to coalescence and ultimate fragmentation, and to quantitatively measure parameters that would be important in any realistic predictive model.

We devised the "contained fragmenting round" technique that enabled us to load HF-1 cylinders at pressures and strain rates high enough to induce shear banding, but to contain the cylinder in such a manner as to arrest the shear banding process at a level before full fragmentation occurs. We were then able to examine the material metallographically, observe the shear bands, and relate them to the loading histories.

In the following paragraphs, we discuss the experimental techniques in greater detail, examine the results from the experiments, and draw conclusions regarding the shear banding process.

A. Contained Fragmenting Round Experimental Technique

To study the shear banding process through its nucleation-growth-coalescence cycle, we needed a technique somewhat analogous to the plate-impact technique developed at SRI for study of ductile and brittle fracture.[3] The latter technique allows independent variation of tensile stress magnitude and duration, which are the principal parameters controlling the nucleation and growth of ductile voids or brittle cracks. For shear banding, we expected that the principal controlling parameters would be shear strain magnitude and duration, or alternatively, shear strain and strain rate. Thus, we needed a method that would apply a known shear load on a specimen and then remove the load after a known time.

We chose a cylindrical geometry, as it appeared to us to be the simplest geometry for producing shear bands and also the most relevant to the fragmenting round problem.

The geometry is shown schematically in Figure 1. Dimensions and other parameters are given in Table 1. The specimen material is HF-1 steel, a material that has been used in naturally fragmenting rounds as well as in previous shear banding studies. The HF-1 steel tubes were provided by the Naval Surface Weapons Center.[4]

The specimen tube is filled with a high explosive and detonated through an H.E. cone to produce a relatively planar detonation wave through most of the tube. The high internal pressures from the H.E. deform the tube outward, inducing a state of strain that has a large compressive radial component, a large tensile hoop component, and a negligible axial component (ignoring end effects). We thus have a state of high shear strain.

The specimen tube is surrounded by a series of concentric containment pipes. The relatively thin-walled, soft Lucite buffer tube allows the specimen tube to expand radially and prevents a sharp high-pressure impact with the main containment tube, which is a very thick-walled mild steel pipe. The outermost lead tube serves as a momentum trap; it is precut in the axial direction to enable it to fly off in several pieces in the radial direction, taking most of the radial momentum with it and reducing the inward-going tensile wave. Because of the thickness of the containment tubes, the specimen tube experiences no radial tensile stress for at least 40 μsec, by which time the internal pressures produced by the H.E. are greatly reduced, due to escape of explosive gases from the open ends of the tube. The lack of radial tensile stresses inhibits brittle fracture, leaving shear banding as the predominant failure mode.

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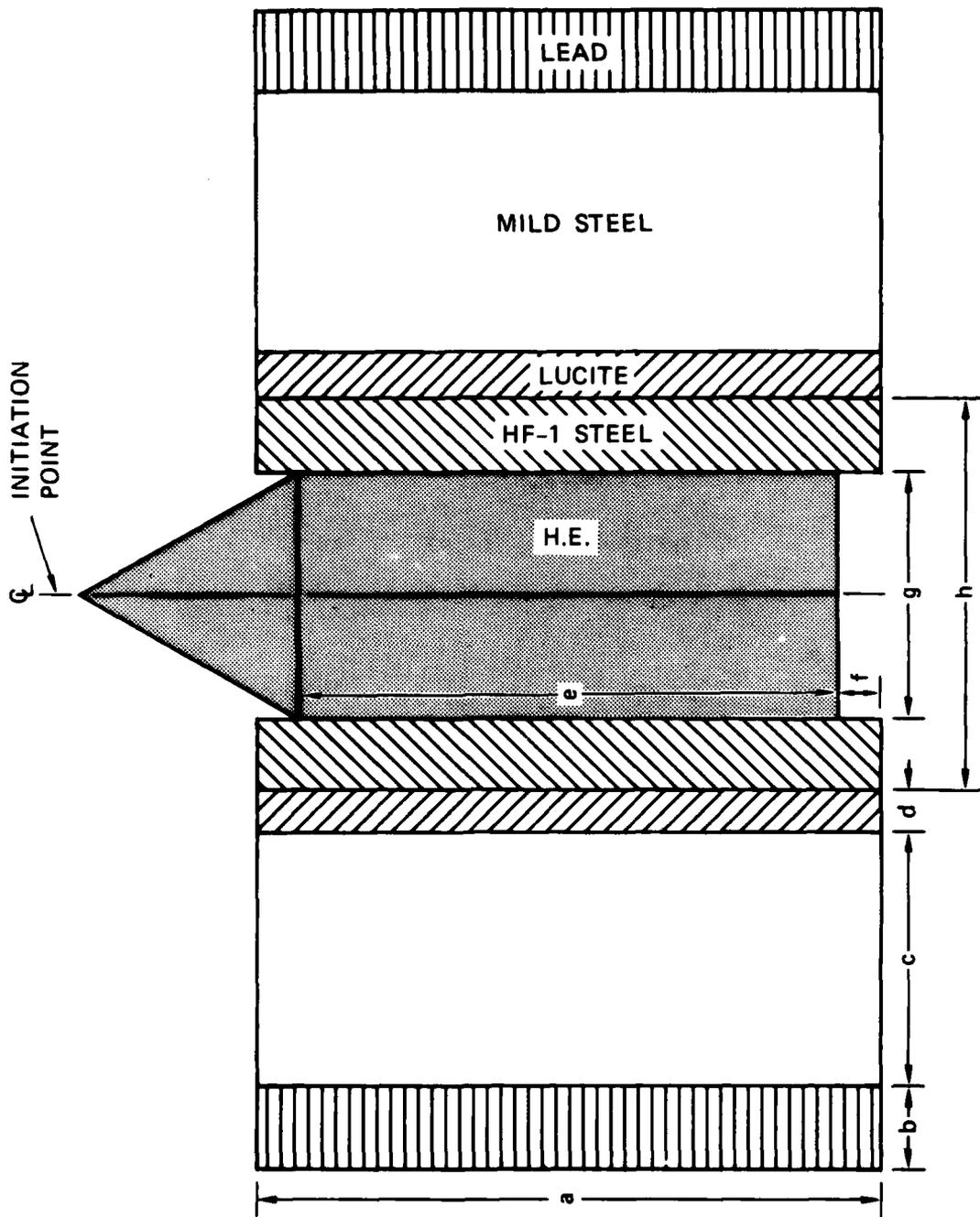
FIGURE 1   CROSS-SECTION DIAGRAM OF CONTAINED FRAGMENTING ROUND EXPERIMENTS
Letters refer to dimensions used in Table 1.

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Table 1

CONTAINED FRAGMENTING ROUND EXPERIMENTAL PARAMETERS

	Shot No.			
	5084-2	5084-3	5084-4	5084-5
Dimensions (cm)				
HF-1 steel tube				
I.D. (g)	7.68	7.68	7.68	7.68
O.D. (h)	12.07	12.07	12.07	12.07
Length (a)	18.73	19.05	19.05	18.42
Lucite sleeve thickness (d)	1.27	1.27	2.86	1.27
Steel containment pipe wall thickness (c)	7.94	7.94	6.35	7.94
Lead thickness (b)	2.54	2.54	2.54	2.54
Explosive column				
Length (e)	17.78	13.97	14.60	12.06
Setback (f)	0	2.54	2.22	2.22
Explosive parameters				
H.E. used	PETN/ μ-spheres[†]	Nitro-methane	PETN/ μ-spheres	PETN/ μ-spheres
Explosive density (gm/cm^3)	0.98	1.135	1.05	1.34
Expected detonation				
Velocity (mm/μsec)	5.45	*	5.75	6.9
Pressure (kbar)	75	*	90	160

Note: (a)-(h) refer to dimensions shown in Figure 1.

*Data uncertain due to impurities in nitromethane, but was similar to that in Shot No. 5084-2.

†Pentaerythritol tetranitrate ($C_5H_8O_{12}N_4$)/hollow plastic micro-spheres manufactured by Emerson & Cuming, Inc., Canton, Mass.; - 2% μ-spheres by weight.

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The strain and strain rate to which the specimen tube is subjected can be varied by changing the explosive mixture and the thickness of the Lucite buffer tube. A higher pressure detonation will induce higher strain rates in the specimen tube, and a thicker buffer will allow the specimen tube to attain larger strains. However, the controlling parameters are not orthogonal with respect to the material response parameters. For example, a higher detonation pressure will produce larger strains even if the buffer tube thickness remains the same. Furthermore, there is a complicated pattern of stress waves and reflections while the various containment tubes are "ringing up" to their peak pressures and then unloading. Hence, the loading history of the specimen tube cannot be predicted easily without the aid of computer simulations.

B. Experimental Results - Shear Band Nucleation

After running several computer simulations (using Cylindrical PUFF), and one proof test using a 4142 steel specimen tube (so as not to deplete our limited supply of HF-1 steel) to establish the geometry of the experiment, we performed four contained fragmenting cylinder experiments with HF-1 steel, as shown in Table 1.

The first two experiments (Shots 2 and 3) resulted primarily in incipient shear band formation, while the latter two (Shots 4 and 5) resulted in substantial shear band growth and partial fragmentation. The specimen tube recovered from Shot 2 is shown in Figure 2. The partially fragmented shear lip and substantial shear cracking at the upper end of the tube is caused by the large axial strain resulting from the H.E. extending axially to the very edge of the tube. Such end effects were eliminated in subsequent shots by setting the H.E. back from the tube edge. The tube recovered from shot 3, shown in Figure 3, exhibits a radial deformation that is greater near the center

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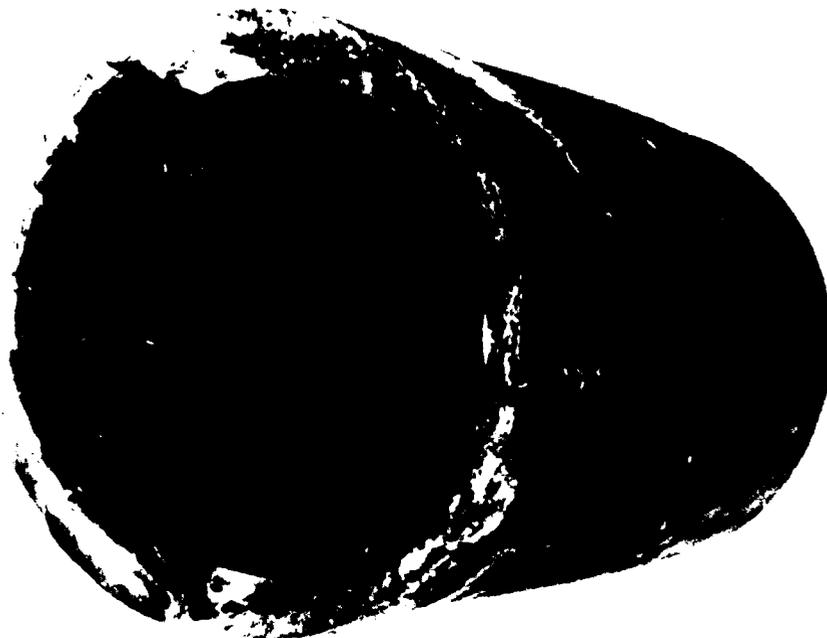
(b) TOP VIEW

(a) SIDE VIEW

FIGURE 2   HF-1 STEEL CYLINDRICAL TUBE DAMAGED IN SHOT 5084-2

MP-5084-2

10
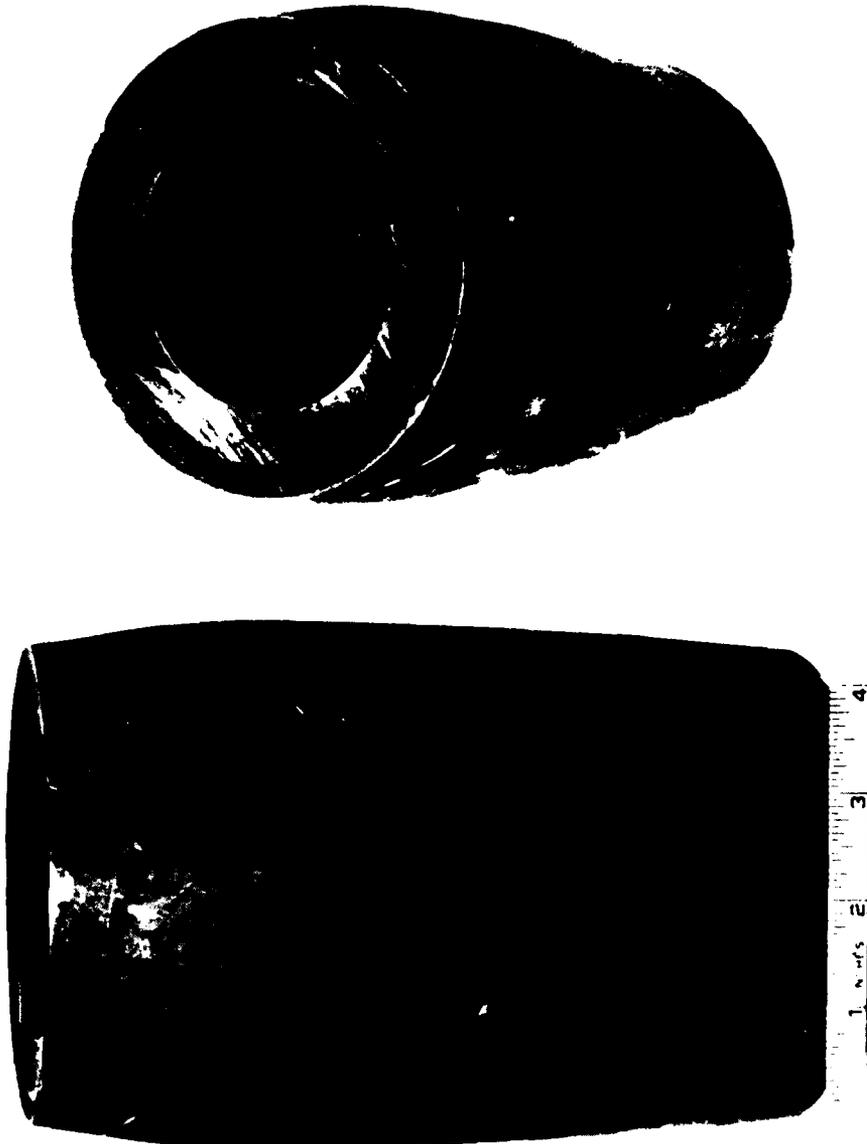
(a) SIDE VIEW

(b) TOP VIEW

MP-5084-3

FIGURE 3   HF-1 STEEL CYLINDRICAL TUBE DAMAGED IN SHOT 5084-3

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of the tube than at the ends because of the explosive set-back and because the internal pressure at the center remains high for a longer period of time.

The wall thickness and diameter of the recovered tubes were measured as a function of axial position and plotted to show the final shapes as shown in Figure 4. The equivalent plastic strain undergone by the tube at different axial positions was then calculated assuming negligible axial strain. A definition of equivalent plastic strain and details of this calculation can be found in Appendix A. Equivalent plastic strain was selected as a parameter relevant to the shear banding process that could be compared with computer simulations. For Shot 3, the strain at the position of maximum deformation was $\approx 28\%$.

Other than the edge effect damage discussed above, the only visible sign of material failure in Shots 2 and 3 was the presence of minute axial striations on the inner tube surface, as can be seen in Figures 2(b) and 3(b). These striations appear after the tubes have deformed to an equivalent plastic strain of $\approx 15\%$ and often extend for an inch or more along the cylinder axis. We hypothesized that these striations are the sites for subsequent shear banding. To test the hypothesis, we made a cut in the tube recovered from Shot 3, perpendicular to the cylinder axis near the point of maximum deformation. Photomicrographs of the polished and etched surface are shown in Figure 5, and these pictures clearly show the incipient stages of shear band formation. The first stage appears as surface striations, as the material experiences non-uniform radial deformation. Further radial expansion causes shear motion to concentrate at the striations. Once this nonuniform shear deformation has begun, the local material becomes hotter and weaker, and the shear band continues to propagate into the material along the 45° plane of maximum shear stress, provided that the internal pressures remain high enough to continue radial expansion. The material along the

12

SHOT 5084-2

SHOT 5084-3
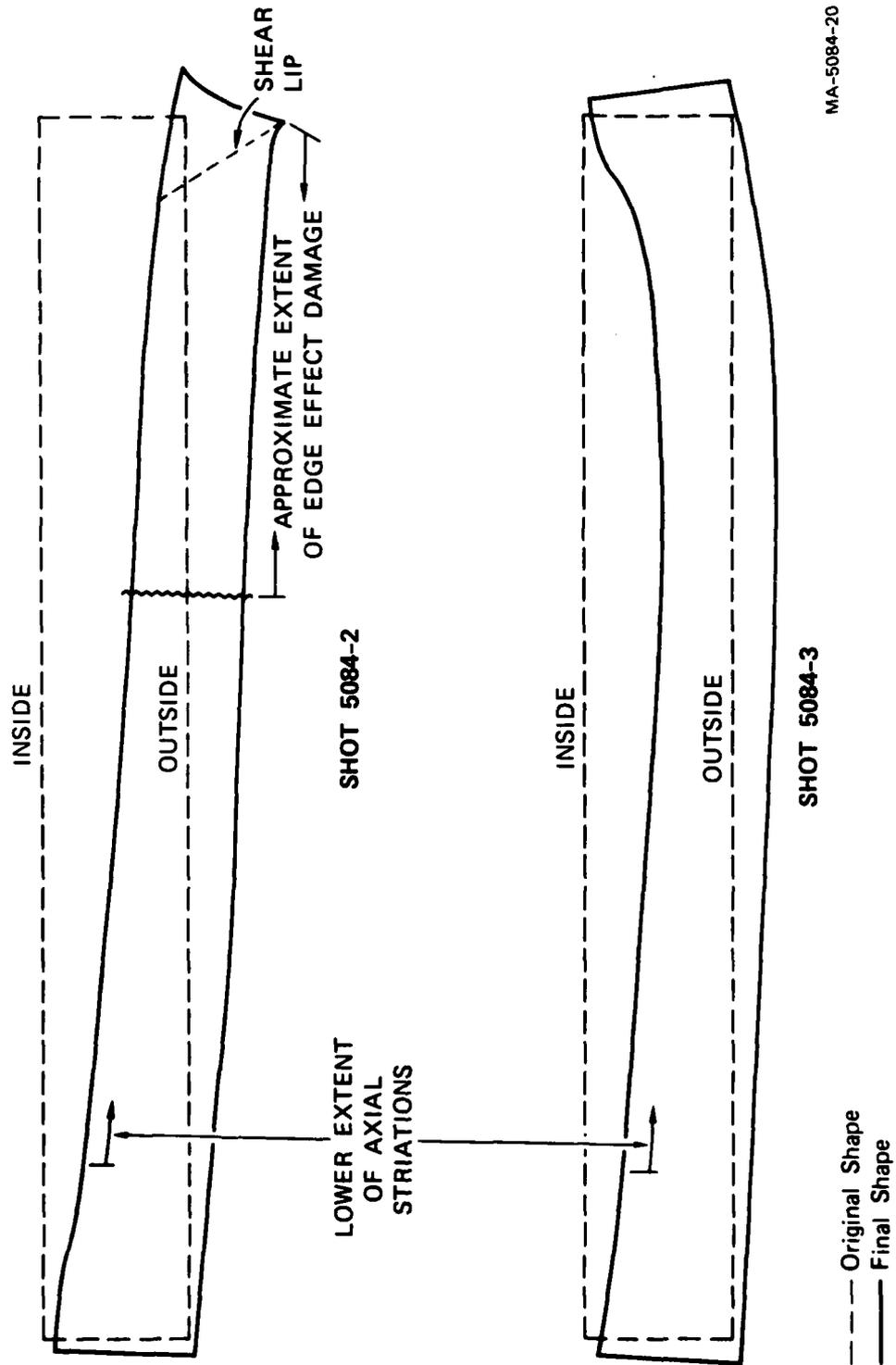
FIGURE 4   OUTLINES OF HF-1 STEEL TUBE WALLS BEFORE AND AFTER DEFORMATION
IN CONTAINED FRAGMENTING ROUND EXPERIMENTS

MA-5084-20

13

(a)

(b)

0.5 mm
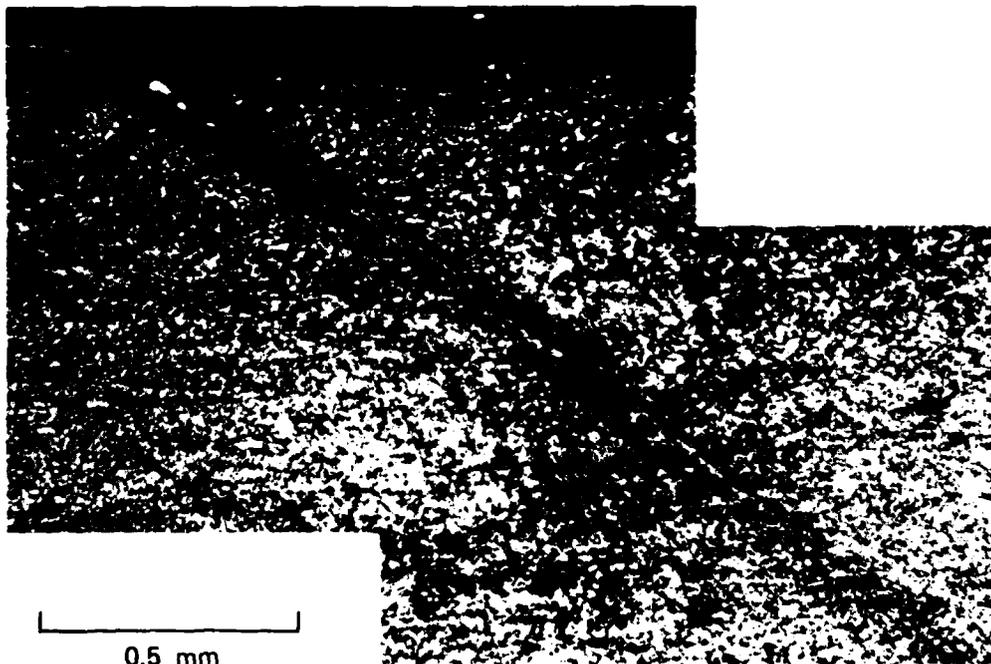
(c)

MP-5084-15

FIGURE 5   INCIPIENT SHEAR BAND FORMATION IN HF-1 STEEL SHOWING
(a) SURFACE DIMPLING, (b) INITIAL SHEAR MOTION AT A DIMPLING
SITE, AND (c) SUBSEQUENT GROWTH

shear band has a reduced tensile strength, so that subsequent tensile stresses can produce a crack in the shear-banded zone, as seen in Figure 5(c).

Other cuts were made in the HF-1 steel tubes recovered from Shot 3 and from the regions of Shot 2 unaffected by edge effect damage, but metallographic observation revealed no damage other than the type shown in Figure 5--incipient shear band formation and band growth to a depth of only a few millimeters. Thus we can conclude that in this geometry and at the strain rates produced by H.E. detonation pressures on the order of 100 kbar, shear bands begin to nucleate at equivalent plastic strains greater than $\approx 15\%$, but have not grown to significant size (the size where coalescence or fragmentation may take place) at equivalent plastic strains of 28%.

C. Experimental Results - Shear Band Orientation, Growth, and Coalescence

Contained fragmenting round experiments 4 and 5 resulted in much greater levels of damage to the specimen tube than the earlier shots. This was caused by higher detonation pressures of the H.E. (Shot 5) or by thicker Lucite buffer tubes (Shot 4). In both cases the containment system was not strong enough to keep the specimen tube intact. As a result, the HF-1 steel fragmented into many relatively large pieces, most of which were recovered. Figure 6 is a photograph of recovered fragments from Shots 4 and 5; recovered fragment size distributions are shown in Figure 7. (The equivalent radius (R_E) is the radius of a sphere having the same mass as the fragment.)

Figure 8 depicts cylindrical tube elements containing shear bands of the three possible orientation types for cylindrical symmetry. For every shear band, the material on one side of the band slips with respect to the material on the other side. If a shear band intersects a material surface, part of the slip plane is exposed above the surface (the shaded regions in Figure 8). We define l as the length of this exposed region, or, more generally, as the length of the band in the direction perpendicular to the slip direction. If we cut the material along a plane perpendicular to the shear band length, we see the band edge-on and can define the parameters d as the depth of the band in the slip direction, or the average depth of the material on opposite sides of the band, and B as the shear displacement or slip magnitude at the surface. (B corresponds to Burger's vector in atomic dislocation theory.)

For orientation type 1, l is in the axial direction; for type 2, l is in the circumferential direction; and for type 3, l is in the radial direction. Within each orientation type, there are two possible directions--90° with respect to each other--representing planes of shear strain symmetry. All six of these directions represent slip along planes of maximum shear strain. For a radially expanding

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(a)
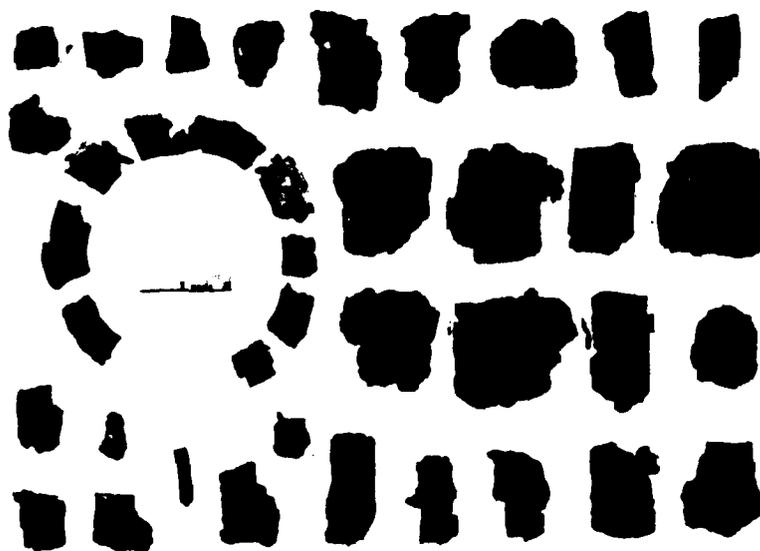
(b)

MP-5084-4

FIGURE 6   HF-1 STEEL FRAGMENTS RECOVERED FROM (a) SHOT 5084-4
AND (b) SHOT 5084-5

Arrow shows fragment used for quantitative shear band study.

17
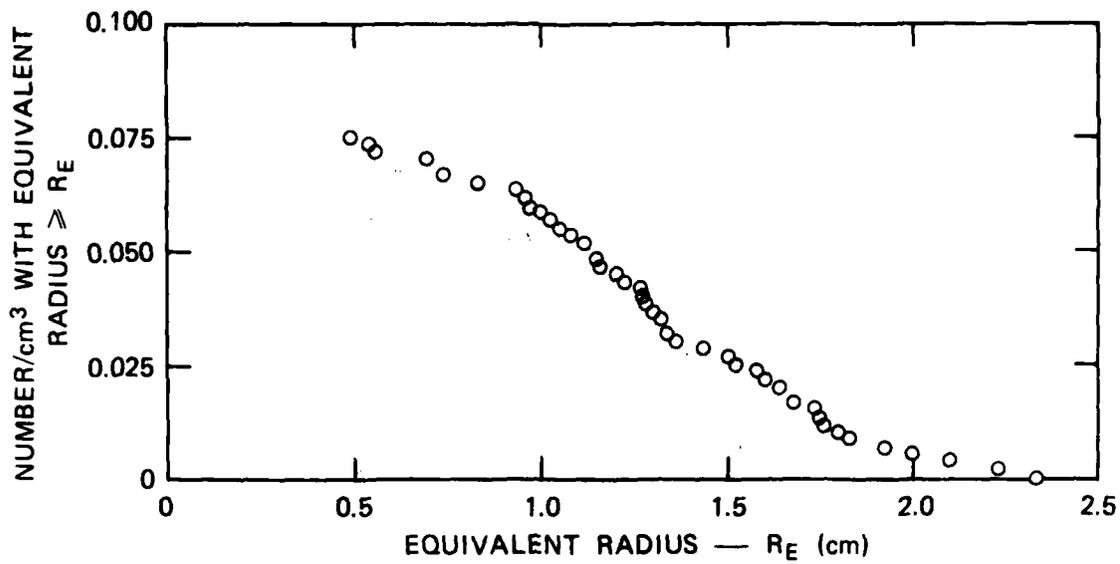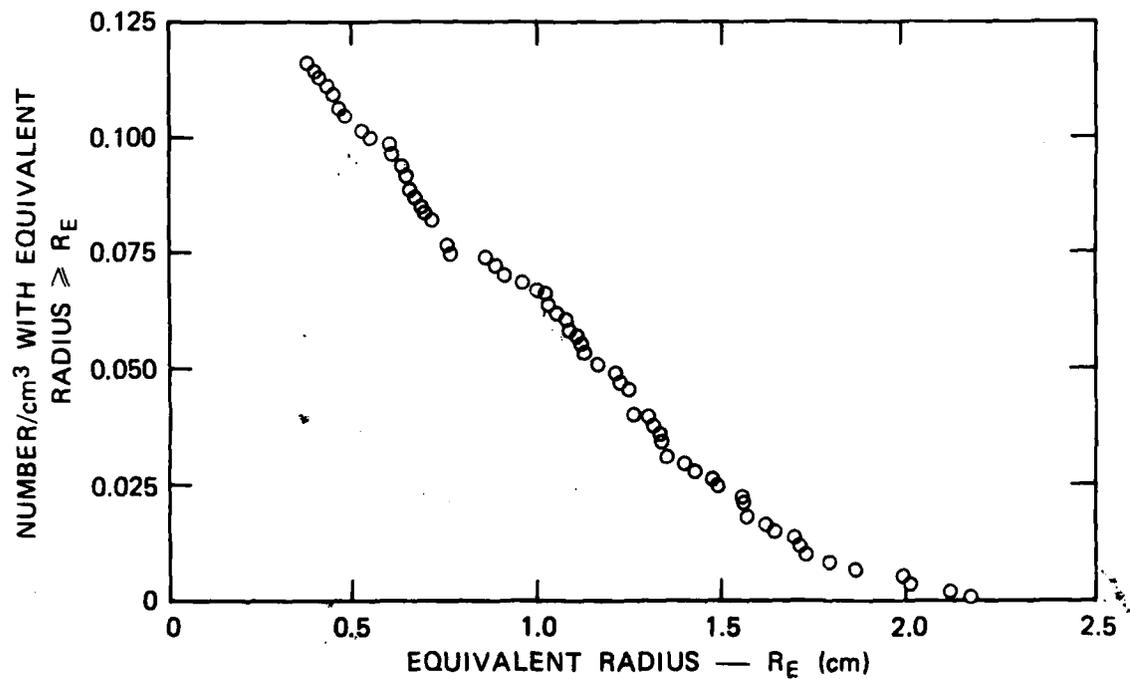
FIGURE 7 FRAGMENT SIZE DISTRIBUTIONS FROM (a) SHOT 5084-4 AND (b) SHOT 5084-5

MA-5084-13

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ORIENTATION TYPE 1

(a)

ORIENTATION TYPE 2

(b)
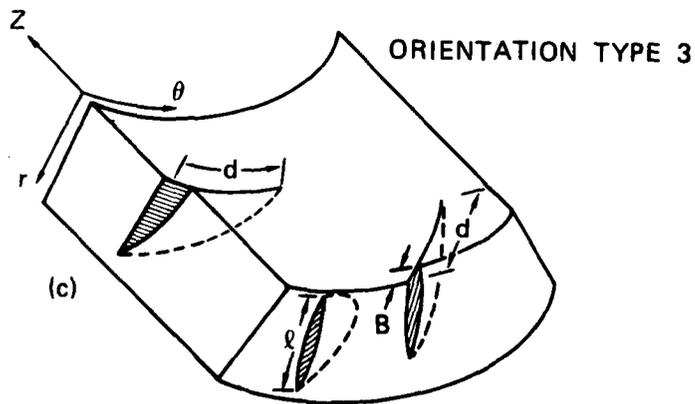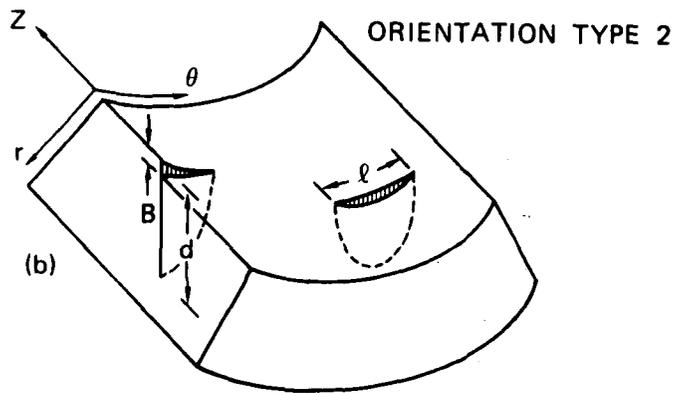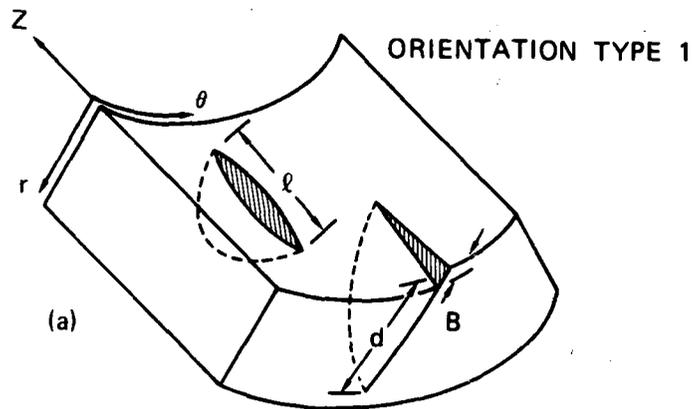
ℓ — Length in Direction
    Perpendicular to
    Slip Motion

d — Depth Along 45°
    Slip Plane

B — Shear Displacement
    Along Slip Plane

Z — Axial Direction

r — Radial Direction

θ — Circumferential
    Direction

ORIENTATION TYPE 3

(c)

MA-5084-21

FIGURE 8   GEOMETRY AND NOMENCLATURE FOR SHEAR BANDS IN CYLINDRICAL
ROUND EXPERIMENTS

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cylindrical tube, the largest of these shear strains are in the plane perpendicular to the axial direction, so shear bands of orientation type 1 are predominant in the fragmenting cylinder experiments. Type 1 accounts for all of the shear band damage in Shot 4, as well as most of the damage in Shot 5.

Figure 9 shows a band[*] that, for much of its length, has not separated in tension, although the material on opposite sides of the transformed band has slipped distances that are orders of magnitude greater than the $\approx$ 5-μ band thickness. Large displacements, therefore, do not in themselves cause the shear band to crack. Figure 10, on the other hand, shows a band that has almost completely separated. This separation occurs sometimes entirely within the transformed band, some-time adjacent to the band but parallel to it, and sometimes across the band at a slight angle. Whether a crack lies entirely within a shear band is probably determined by the time elapsed between band formation and subsequent tensile failure. If the hot shear band has had time to be quenched by the surrounding material and thus regain some of its strength, the crack will more likely lie partially outside the band. In any case, shear cracking is seen only in the vicinity of shear banding in the fragmenting cylinder experiments, and so for the quantitative damage assessment discussed below, shear bands and shear cracks are considered the same.

The fragments recovered from Shot 4 were created almost entirely by the shear banding process; the lateral edges of the fragments were all shear bands of orientation type 1. The fragments recovered from Shot 5, on the other hand, showed shear band damage only in the region from the inner surface to about halfway through the wall thickness, and showed

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[*]The band appears white after etching because the steel has been metal-lographically transformed by the heat generated in the band formation process.

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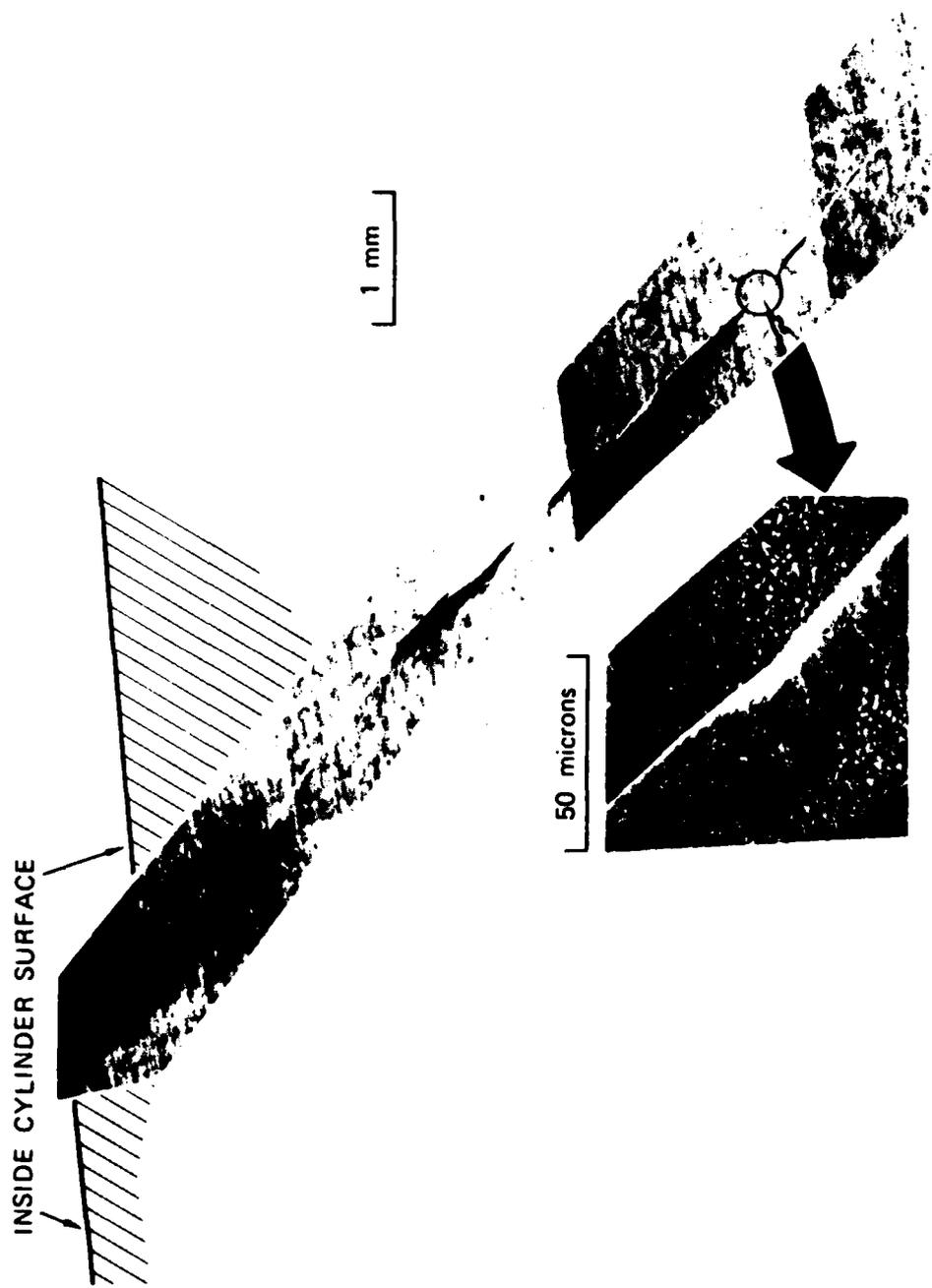
FIGURE 9   DETAILED PHOTOMICROGRAPH OF CRACK 9A ON CUT 7, WITH INSET SHOWING HIGH MAGNIFICATION OF SHEAR BAND REGION

MP-5084-8

INSIDE CYLINDER SURFACE

1 mm

50 microns

21

INSIDE CYLINDER SURFACE

1 mm

MP-5084-7

FIGURE 10   DETAILED PHOTOMICROGRAPH OF CRACK 3A ON CUT 4, SHOWING VARIOUS
RELATIONSHIPS BETWEEN SHEAR BANDING AND SUBSEQUENT CRACKING

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primarily brittle fracture the remainder of the way toward the outer surface.* We therefore chose to study in detail a fragment from Shot 4 and selected the one denoted by an arrow in Figure 6, as it appeared to contain a large number of shear bands that intersected the inner surface.

Photographs of the fragment from two different directions are seen in Figure 11. The two directions A and B highlight the shear bands that intersect the inner surface at the two different 45° planes. For identification purposes, the shear bands are labelled by a number and a letter (e.g., band 1B is the band numbered 1 that is visible from direction B). To allow examination of internal damage, the fragment was cut into thin layers perpendicular to the axial direction along the lines shown in Figure 11. The cut surfaces were then polished, etched, and examined metallographically. Figure 12 is a composite photomicrograph of the surface of cut 4, showing clearly how the shear bands visible on the inner surface extend into the material, and also revealing internal shear bands.

Examination of the composite photomicrographs taken from each of the eight cuts revealed that more than two-thirds of the shear bands intersected the inner surface of the cylinder. Two bands extended from the interior to the outer surface, and the remainder lay entirely within the interior. We believe that the bands originated at their innermost edge where the shear strains were larger and occurred earlier, and proceeded to grow outward; however, we cannot be certain of that in each case. Although most of the bands in this fragment stopped without interacting with any other bands, some bands did intersect others. Some short bands intersected and stopped at the middle of a longer band, indicating that the short band was formed at a later time than the longer band.

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*This was probably caused by the premature failure of the containment system, which allowed radial relief waves to propagate into the HF-1 tube, causing tensile failures.

(a)
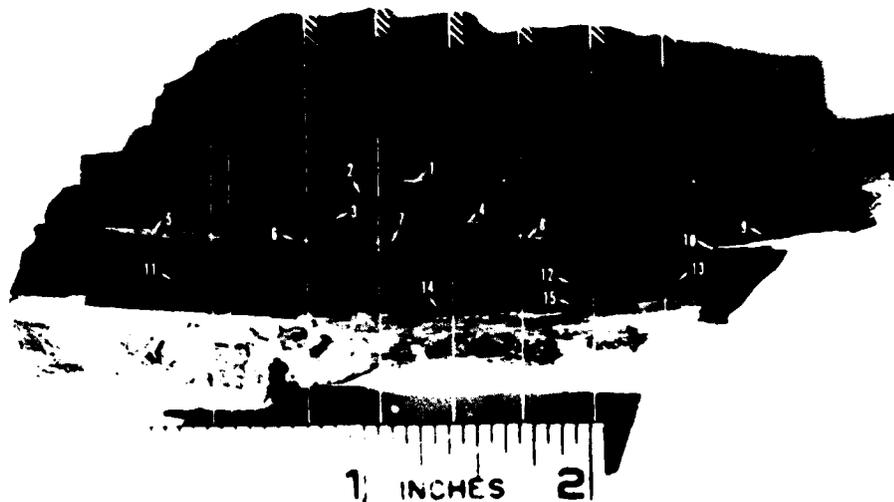
(b)

MP-5084-5

FIGURE 11   HF-1 STEEL FRAGMENT FROM SHOT 5084-4 SELECTED FOR USE IN
QUANTITATIVE SHEAR BAND STUDY, PHOTOGRAPHED FROM (a) DIRECTION
A AND (b) DIRECTION B

24
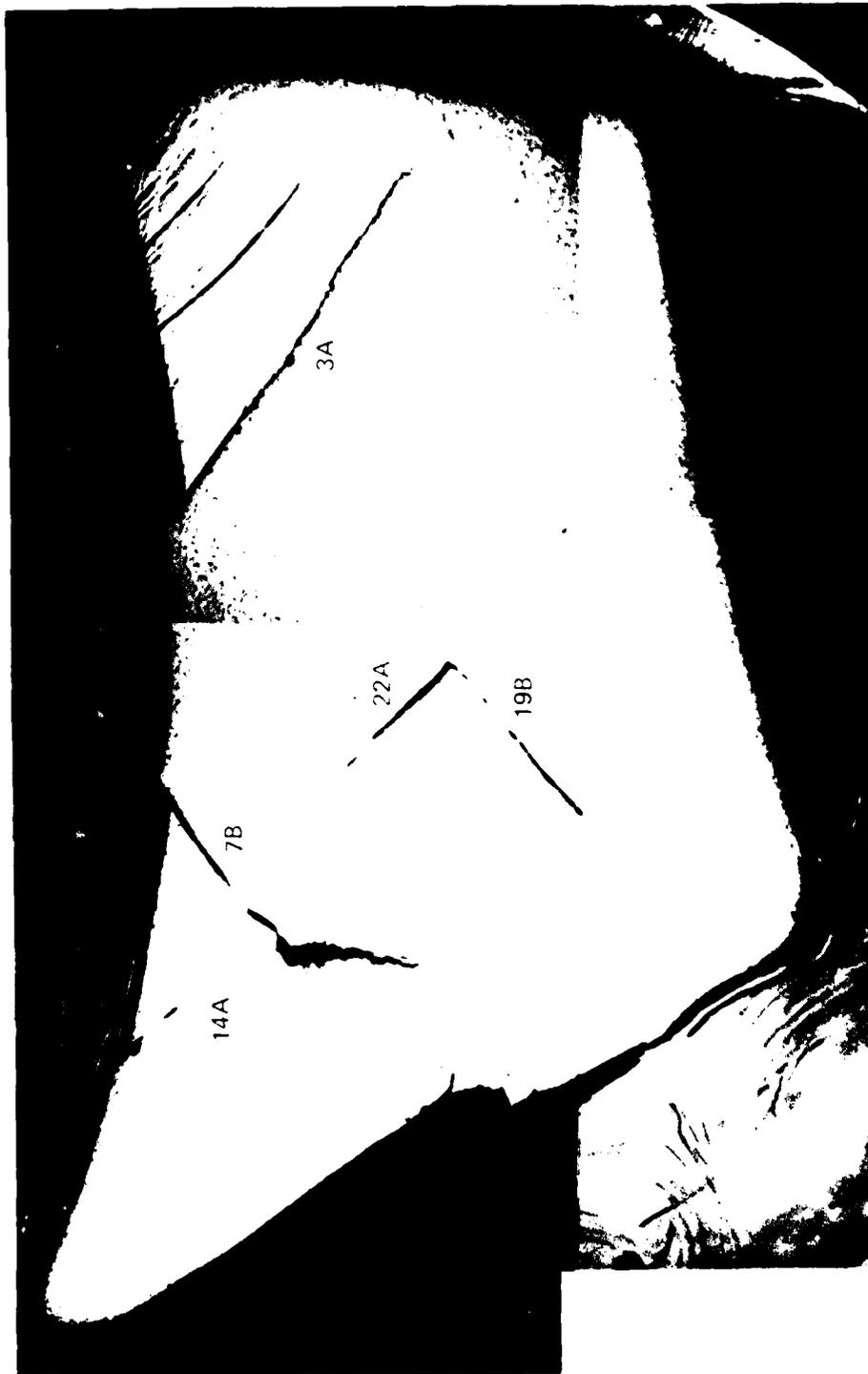
MP-5084-6

FIGURE 12   COMPOSITE PHOTOMICROGRAPHS OF CUT 4, SHOWING VARIOUS POSITIONS AND
ORIENTATIONS OF SHEAR BANDS AND CRACKS WITHIN HF–1 FRAGMENT

Note shear bands 14A and 7B intersect to form tensile crack.

Other intersecting bands, such as bands 14A and 7B in Figure 12 appeared to grow simultaneously and interact to form a tensile crack beginning at their point of intersection.

The depth (d) of all of the bands revealed on the eight composite photomicrographs was measured, yielding the distribution shown in Figure 13. Since the cuts did not necessarily intersect the bands at their point of maximum depth, the depth values measured are only lower bounds. If we examined a greater number of bands, a statistical transformation similar to that used for tensile failure analysis[4] could be used to determine the actual distribution.

The surface shear displacement (B) for all the bands intersecting the surface was measured and the relationship between d and B for all of those bands is plotted in Figure 14. Three bands that extended over several cuts are identified by special symbols. Most of the points lie close to the line d = 15B, indicating a direct proportion between the shear band depth and surface displacement.

The shear band depths were compared with the length at the inner surface so as to determine the approximate shape of the shear band in its slip plane. The results are plotted in Figure 15, along with the locus of points for a semicircular shear band shape. Again, noting that the depth values are only lower bounds, it appears that ℓ is on the order of twice d. If a shear band is assumed to start growing at a single point, then it appears to grow in depth about as much as it grows in surface length on each side of its starting point. Without an extensive program of successive cycles of grinding, polishing, etching, and photography, we cannot observe the precise shape of the outer edges of the shear bands. However, since d ≈ 15B, we can get an approximate idea by observing the shape of the slip plane exposed above the inner surface in Figure 11 (or in other words, how B varies along the length).

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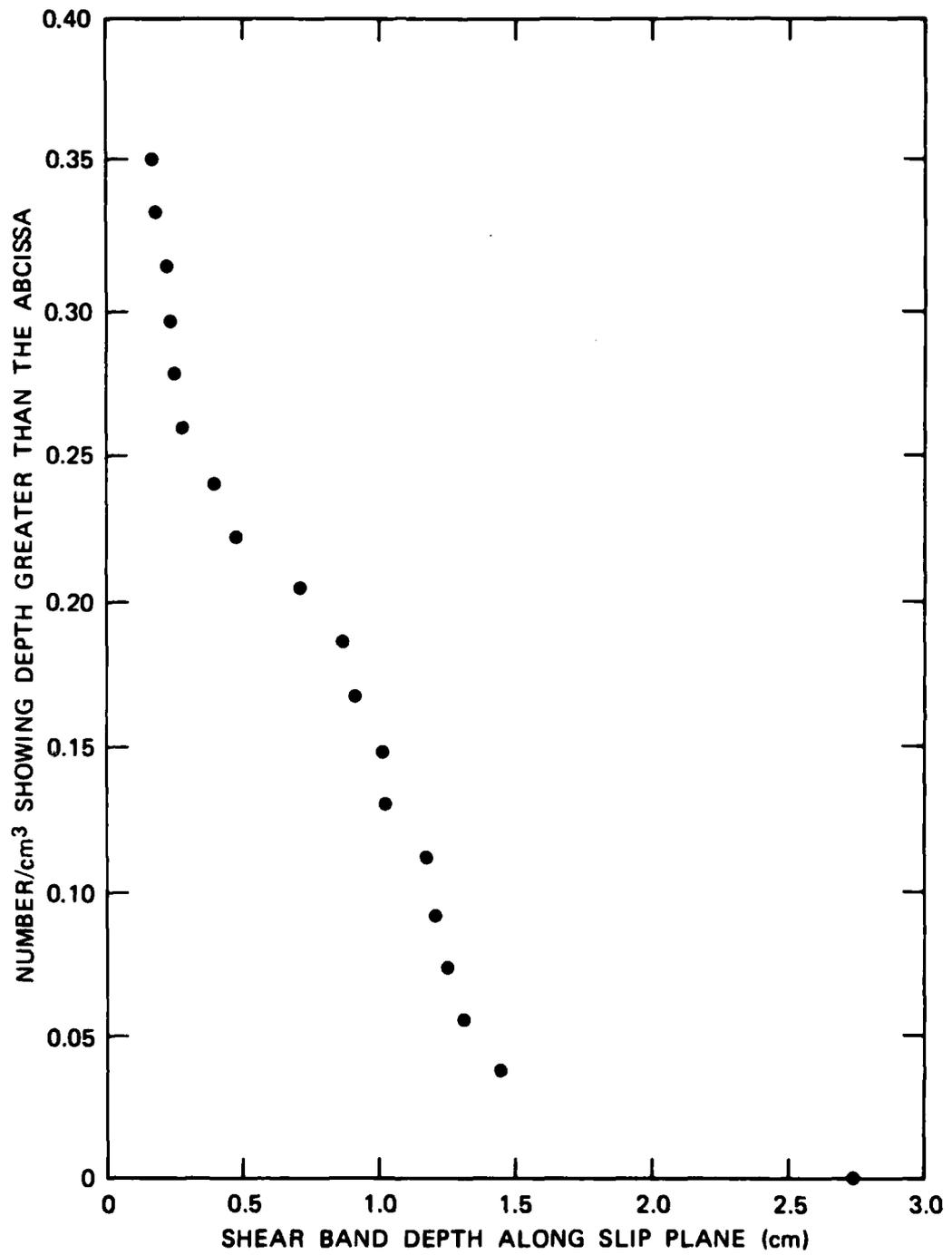
FIGURE 13   DEPTH DISTRIBUTIONS OF SHEAR BANDS IN HF-1 STEEL FRAGMENT
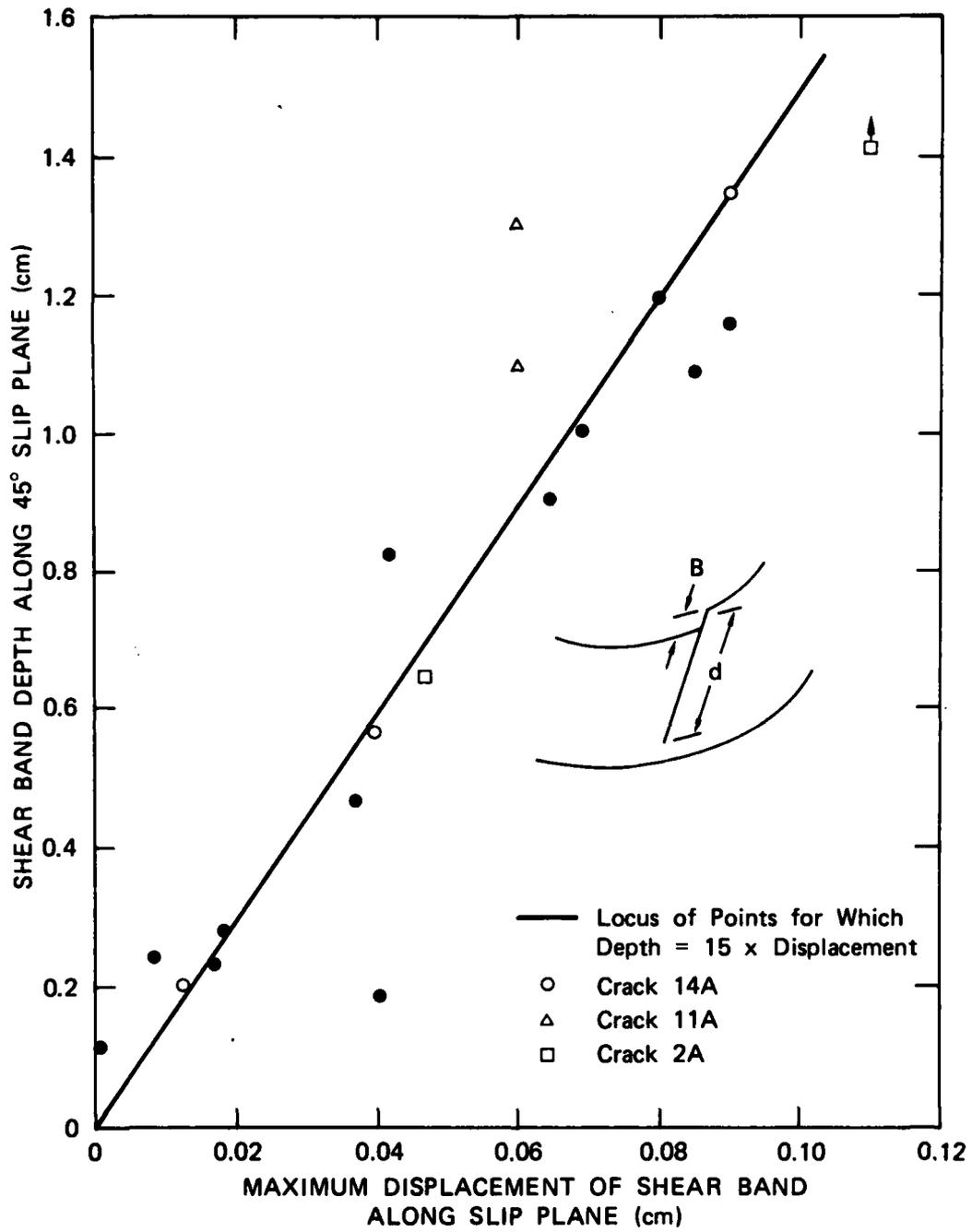
FIGURE 14   SHEAR BAND DEPTH (d) VERSUS DISPLACEMENT (B) DATA

MA-5084-10
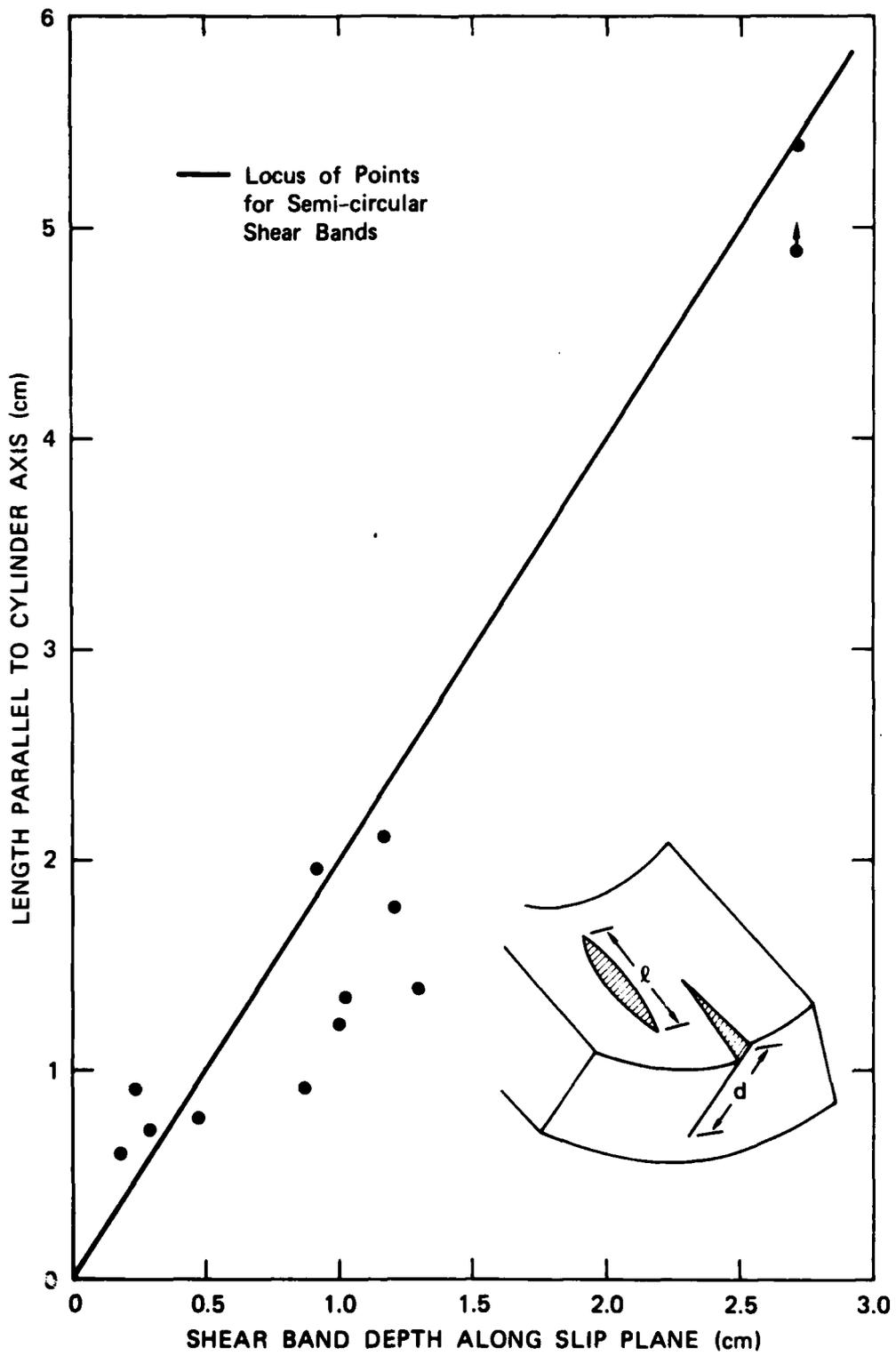
FIGURE 15   SHEAR BAND LENGTH (ℓ) VERSUS DEPTH (d) DATA

MA-5084-11

Using this method, we find that some shear bands appear semicircular, others seem elliptical, and others have more complex shapes. But for computational modeling, it appears that the bands can be represented as semicircles.

D. Conclusions

The above work has lead to the following experimental observations:

- Multiple waves of tension and compression propagate through the thickness of a fragmenting cylinder.
- Shear bands nucleate and grow in proportion to the amount of shear strain.
- Bands usually appear to start at the inner diameter of fragmenting rounds, but often they appear at some depth within the material.
- For bands with large displacement, the radial and axial extent of the bands are comparable; hence, bands are approximately circular.
- The maximum dislocation of material across a band occurs at the center and is 5 to 10% of the radial or axial extent of the band.
- Bands form at 45° to the principal directions; that is, in the directions of maximum shear stress.
- During plastic flow, many small ripples or striations appear on the inner surface at a strain of about 15%. At larger strains, a small number of these striations form shear bands.
- Brittle cracks often form along the bands at a late stage.

In conclusion, the contained fragmenting round technique has proven successful in increasing our knowledge of shear banding phenomena. However more experiments are required to better establish the nucleation criteria and the band growth velocities.

IV   SHEAR BAND MODEL DEVELOPMENT - COMPUTATIONAL PHASE

The computational model for shear banding is a detailed description
of the processes by which shear bands form and grow in a material under-
going large deformation. This section contains a review of the phenomena,
an analysis relating the nucleation and growth to material properties, and
a discussion of additional features needed to implement the model in a
subroutine. A listing of the subroutine and further information about
the parameters are given in Appendix B.

The subroutine containing shear banding acts as a stress-strain
relation in a one- or two-dimensional wave propagation computer program.
The program computes strain increments in a cell of material and then
calls the shear band subroutine to calculate the corresponding stresses.
If a threshold criterion has been exceeded, the subroutine also calculates
the growth of existing bands and the nucleation of new bands, and checks
whether coalescence and fragmentation are occurring. For each cell of
material, there are data arrays containing the size, number, and
orientation of all bands.

The shear band model is intended to account for the experimental
observations listed at the end of the previous section.   In addition to
those features, the model includes rate-dependent elastic-plastic stress-
strain relations.

A.   Derivation of the Model

The model includes nucleation of shear bands, growth of the bands,
coalescence to form fragments, and relaxation of the stresses in accord-
ance with the growth of damage. In the following discussion, the physical
processes of failure by shear banding are outlined. Then each process
is examined and equations are derived to represent the process in the
model.

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Under a high shear strain, materials first deform uniformly, following approximately the usual assumptions of homogeneity and isotropy. But as the plastic strains become large, inhomogeneities may develop. At these inhomogeneities, a slip like a small crack can occur parallel to the direction of maximum shear stress and strain. With continued straining, the slipped region or shear band grows in a circular fashion. That is, the growth rate in the direction of the shear slip is approximately equal to that in the direction perpendicular to the slip. Following nucleation of the bands, growth can occur until the bands coalesce. On a cross section the band is observed as a narrow strip of highly deformed material. Probably the strip is heated approximately to melting during growth of the band. At the end of growth, the band material remains welded together, but with a reduced tensile strength. Hence, subsequent tensile stresses in the material will fracture in or along the band.

1. Nucleation

The nucleation process includes several features: a threshold for nucleation, the rate of nucleation, the size of bands at nucleation, and the orientation of the bands. When the plastic shear strain exceeds a threshold value, nucleation begins. In the HF-1 material there is a threshold at about 15% at which small irregularities appeared in the surface. But since significant slipping begins at 30%, this value of plastic strain serves as the threshold criterion. Thus, the threshold is an observed quantity and probably varies from one material to the next.

The derivation conducted by Seaman et al.[2] indicated that the rate should be proportional to the square of the yield strength and to the applied strain rate; the resulting calculations correlated fairly well with the measured fragment size distributions. In that study the orientation of the shear bands was not considered, so nucleation referred

to the sum of the bands in all directions and the shear strain was a scalar (undirected) quantity.

Here a nucleation rate function is proposed to account for the observed relationships to plastic shear strain and yield strength and to provide an orientation dependence for the shear band process. The major assumption in the following proposed expression is that the nucleation rate in a direction is proportional to the plastic shear strain in the same direction.

$$\frac{dN_{\varphi\theta}}{dt} = C_n F_n \left(\frac{1}{E_{cr}} \frac{dE^p}{dt} \right)^2 \frac{d\epsilon_{\varphi\theta}^{ps}}{dt} \tag{1}$$

Here E_{cr} is a critical energy, taken to be the melt energy; dE^p/dt is the rate of increase of plastic strain energy; $N_{\varphi\theta}$ is the number of shear bands per unit volume in the $\varphi\theta$ direction;[*] and $\epsilon_{\varphi\theta}^{ps}$ is the plastic shear strain in the same direction. C_n is a coefficient with the units of time squared, and F_n is a factor representing the fraction of solid angle assigned to the $\varphi\theta$ orientation. Orientation effects are treated in more detail later. The factor dE^p/dt is proportional to the product of the shear strain rate and the yield strength; hence, the nucleation rate is proportional to the cube of the strain rate.

The plastic strain energy is calculated as follows.

$$dE^p = V \sum \sigma'_{ij} \, d\epsilon_{ij}^p \tag{2}$$

where V is specific volume,

---

[*] Here φ is the polar angle and θ is the angle of rotation about the pole.

σ'_{ij} is the deviator stress in the ij direction, and

$d\epsilon^p_{ij}$ is the plastic strain increment in the ij direction.

The plastic strains in any orientation are given by the flow rule

$$d\epsilon^p_{ij} = \frac{\sigma'_{ij}}{\bar{\sigma}} d\bar{\epsilon}^p \tag{3}$$

where $\bar{\sigma} = \sqrt{1.5}\sqrt{[(\sigma'_{11})^2 + (\sigma'_{22})^2 + (\sigma'_{33})^2 + 2(\sigma'_{12})^2]} \tag{4}$

$$d\bar{\epsilon}^p = \sqrt{1.5}\sqrt{[(d\epsilon^p_{11})^2 + (d\epsilon^p_{22})^2 + (d\epsilon^p_{33})^2 + 2(d\epsilon^p_{12})^2]}. \tag{5}$$

With the $\sqrt{1.5}$ factor in the definition of $d\bar{\epsilon}^p$, $d\bar{\epsilon}^p$ is the shear strain in any geometry where two strain components are equal. The plastic shear strains in the six orientations, $d\epsilon^{ps}_{ij}$, are obtained by using for σ'_{ij} in Eq. 3 only the shearing component of the deviator stress acting on each of the six planes. These shear stress components are listed in Table 2.

Table 2

SHEAR STRESS COMPONENTS ACTING ON SHEAR BAND PLANES

Direction[a] Index	Direction[c]	S.B. Type[b]	Shear Stress Component
1	axial	–	σ'_{12}
2	radial	–	σ'_{12}
3	circumferential	–	none
4	45° between axial and radial	2	$\tfrac{1}{2}(\sigma'_{11} - \sigma'_{22})$
5	45° between axial and circumferential	3	$\tfrac{1}{2}\sqrt{(\sigma'_{11} - \sigma'_{33})^2 + 2\,\sigma'^2_{12}}$
6	45° between radial and circumferential	1	$\tfrac{1}{2}\sqrt{(\sigma'_{22} - \sigma'_{33})^2 + 2\,\sigma'^2_{12}}$

Notes

a The direction index indicates one of 6 stress directions.
b Shear band orientation type as shown in Figure 8.
c Orientations used in a two-dimensional, axisymmetric code (TROTT).

Shear bands may appear in many orientations and with many sizes. However, for one- and two-dimensional problems, we have chosen six specific orientations to represent the continuum of all orientations. The orientations provided are for bands normal to three perpendicular coordinate directions and for three more at 45° to the first three. The latter three lie in the x-y plane, x-z plane, and y-z plane, respectively. For each of these orientations a fraction of all possible orientations is assigned. Each of the first three orientations represents two points on the unit sphere (for example, $x = \pm 1$) whereas each of the 45° orientations represents four points. Therefore, it seemed reasonable to weigh the six points with factors, $F_n = 1/9$, $1/9$, $1/9$, $2/9$, $2/9$, and $2/9$, respectively.

For each orientation there is a set of size groups consisting of a number of bands per cubic centimeter N_i with a radius R_i. Initially these size groups represent the continuous distribution

$$N_g = N_o \exp(-R/R_n) \qquad (6)$$

where N_g is the number of bands/cm^3 greater than R

N_o is the total number

R_n is a nucleation size parameter.

This continuous distribution is divided into size groups such that the ith group contains ΔN_i bands given by the following formula:

$$\Delta N_i = \dot{N}\Delta t \left[\exp(-R_i/R_n) - \exp(-R_{i+1}/R_n)\right] . \qquad (7)$$

By using size groups that are discrete in size and orientation it is possible to represent growth and nucleation processes that are strongly dependent on shear band size and plastic strain tensors that vary greatly with orientation.

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2. Growth

The growth of a shear band refers to the increase in radius of the slipped area. Following Seaman, et al.,[2] we have assumed that the velocity of the shear band is proportional both to its radius and to the plastic shear strain rate; that is,

$$\frac{dR_i}{dt} = C_G \left[\frac{d\epsilon^{ps}_{\varphi\theta}}{dt} R_i \right] \tag{8}$$

where C_G is a growth coefficient, $d\epsilon^{ps}_{\varphi\theta}/dt$ is the plastic shear strain rate on the $\varphi\theta$ plane, and R_i is the radius of the ith band in a size distribution. The growth in Eq. (8) is limited by two additional requirements: The velocity cannot exceed the shear wave velocity, and the total distortion of all the bands in the $\varphi\theta$ orientation cannot exceed the total plastic shear strain on the orientation. The first of these requirements is simply stated as

$$R_2 \leq R_1 + V_{max} \Delta t$$

where V_{max} is the shear wave velocity. The second requirement is based on a restatement of the Orowan equation used in dislocation dynamics. The appropriate expression here is

$$\frac{d\epsilon^p}{dt} = \frac{d}{dt} (\pi \sum_i N_i R_i^2 B_i) \tag{9}$$

where B_i is the average shear slip over the area πR_i^2 of the band. If N_i (the number of shear bands) is replaced by the band edge length $n_i = 2\pi R_i N_i$, and N_i is presumed constant during differentiation, the usual Orowan equation is obtained. Over a finite time interval and for

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a fixed number of bands, Eq. 9 takes the form

$$\Delta\epsilon^p = \pi\Sigma N_i(B_{2i}R_{2i}^2 - B_{1i}R_{1i}^2) \tag{10}$$

where subscripts 1 and 2 indicate quantities before and after the time step. If $\Delta\epsilon^p$ from Eq. 10 exceeds the total imposed increment in plastic strain, then R_2 is reduced to meet the requirement.

From the experimental data of our fragmentation tests 4 and 5, the dislocation in a circular shear band appears to be 5 to 10% of the radius. That is,

$$B = bR \quad . \tag{11}$$

Thus, the dislocation increases in proportion to the radius. Eq. 10 then takes the form

$$\Delta\epsilon^p = \pi b\Sigma N_i(R_{2i}^3 - R_{1i}^3) \quad . \tag{12}$$

3.   **Effects of Damage**

Damage caused by shear bands leads to a relaxation of the stresses as well as to eventual separation of the material into fragments. The relaxation effect is produced when damage reduces the area across which stress can be transmitted. The average stress σ over the total or gross section is then related to the stress σ^s in the intact or solid material through a damage quantity D. In tension the relation is

$$\sigma = \sigma^s(1-D) \quad . \tag{13}$$

The gross section A_g is composed of a solid area and a damaged area $A_g(1-D)$. The next step is to derive an appropriate expression for D that will approach 1.0 for full fragmentation and account for the anisotropy of damage.

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To determine an appropriate damage function, we examine two quantities: the total plastic strain taken by the bands and the fraction of the material that has fragmented. The total plastic strain in any orientation is, according to Eq. 12,

$$\epsilon^p = \pi b \sum_i N_i R_i^3 \qquad (14)$$

When the material breaks into fragments, the volume of fragments is

$$V = T_F \sum_i N_i^f (R_i^f)^3 = 1.0 \qquad (15)$$

where T_F is a dimensionless volume factor (about 4.0)

N_i^f and R_i^f are number and radius of fragments in the ith size group.

The numbers of fragments are related to the numbers of bands through the factor β:

$$N_i^f = \beta N_i \quad . \qquad (16)$$

Chunky fragments usually have six or eight sides that were each formed by a band or crack. Because each crack forms a side of two fragments, three or four cracks must be associated with each fragment. Therefore, β equals 1/3 or 1/4. Similarly, the fragment sizes are related to the band sizes through the factor γ:

$$R_i^f = \gamma R_i \quad . \qquad (17)$$

Here R_i^f is defined so that the fragment volume is $T_F (R_i^f)^3$ where T_F is about $4\pi/3$. The cracks forming the fragment sides have about the same area as the sides, so γ is approximately equal to 1. Now Eq. 15

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can be rewritten as a double sum of shear bands over orientations and size groups:

$$V = T_F \beta \gamma^3 \sum_{\theta\varphi} [\sum_i N_{\varphi\theta i} R^3_{\varphi\theta i}] = 1 \ . \tag{18}$$

A comparison of Eqs. 14 and 18 shows that damage appears to be related to NR^3 at all times during the calculation. We will use Eq. 18 as our definition of complete fragmentation ($V = 1$) and let V be the fragmented fraction for values less than full fragmentation. Then the quantity $T_F \beta \gamma^3 \sum_i N_{\varphi\theta i} R^3_{\varphi\theta i}$ is a measure of the damage associated with the bands in the $\varphi\theta$ orientation. We can assume that the damage felt in any direction is proportional to $\cos^2 \psi$ where ψ is the angle between the stress direction and the damage orientation. Then the damage in the x direction is

$$D_{xx} = 3T_F \beta \gamma^3 [T_{xx} + 0.5(T_{xy} + T_{xz})] \tag{19}$$

where

$$T = \sum_i N_i R_i^3 \tag{20}$$

and the subscripts on T identify one of the six orientations considered. The factor 3 in front of the expression for D_{xx} permits D_{xx} to reach 1.0 when V reaches 1.0 with all T's equal. The 0.5 in the expression for D_{xx} is $\cos^2 45°$. The damage quantity in the y and z directions are computed by Eq. 19 with a natural permutation of the subscripts. For the xy direction the damage is

$$D_{xy} = 3T_F \beta \gamma^3 [T_{xy} + 0.5(T_{xx} + T_{yy})] \ . \tag{21}$$

With the foregoing definitions of damage D, we can examine in more detail the relaxation process in Eq. 13. That equation must be modified for compression where we can expect the deviators to reduce with damage but the pressure should not be affected. Toward this end, the gross pressure P is related to the solid stresses σ^s_{ii} for the tensile case. Then the gross stress σ_{ii} is

$$\sigma_{ii} = \sigma^s_{ii} (1 - D_{ii}) \tag{22}$$

$$P = \frac{\Sigma \sigma^s_{ii}(1 - D_{ii})}{3} = \frac{\Sigma \sigma'^s_{ii}(1 - D_{ii}) + P_s[3 - (D_{11} + D_{22} + D_{33})]}{3}$$

$$= \frac{1}{3} \Sigma \sigma'^s_{ii} (1 - D_{ii}) + P_s(1 - D) \tag{23}$$

where $\Sigma D_{ii} = 3D$, P_s is the solid pressure, σ_{ii} is stress on the gross section and σ'^s_{ii} is deviator stress on the intact or solid material. For the compression case a suitable modification of Eq. 23 is obtained by deleting the factor $(1 - D)$ and defining a modified pressure P':

$$P' = \frac{1}{3} \Sigma \sigma'^s_{ii}(1 - D_{ii}) + P_s = \frac{1}{3} \Sigma \sigma^s_{ii}(1 - D_{ii}) + P_s D \tag{24}$$

$$= P + P_s D$$

Then for tension let $P' = P$ and use Eq. 24 to define P' in compression. With these definitions of P', the solid stresses are related to the gross stresses as follows in both tension and compression:

$$\sigma^s_{ii} = \frac{\sigma'_{ii} + P'}{1 - D_{ii}} . \tag{25}$$

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At the beginning of a strain increment calculation, τ and P_s are computed from the existing shear bands and density, and P' is calculated from the old gross stresses, σ_{11} and from P_s for compression. Then the σ_{11}^s are calculated from Eq. 25. At the end of the increment, Eqs. 24 and 25 are again used to obtain the gross stresses.

The criterion for full fragmentation is given in Eq. 18. This criterion is reached simultaneously with the condition that all gross deviator stresses have relaxed to zero.

4. Stress-Strain Relations

In preparation for the stress calculations, the gross stresses from the calling program are transformed to stresses in the intact material using Eq. 13. These stresses are then separated into pressure and deviator components. The pressure is given by the Mie-Gruneisen equation:

$$P = P_H(1 - \frac{\Gamma\mu}{2}) + \Gamma\rho E \qquad (26)$$

where $\mu = \rho/\rho_o - 1$

Γ is Gruneisen ratio

ρ is the current density

ρ_o is the zero pressure density

E is the internal energy

and $P_H = K_1\mu + K_2\mu^2 + K_3\mu^3$ is the pressure on the Hugoniot. K_1 is the bulk modulus, and K_2 and K_3 are coefficients in the Hugoniot expansion.

The deviator stresses are computed from elastic, plastic, work-hardening, rate-dependent relations. For the elastic case the deviator stresses are computed as

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$$\sigma'_{ij} = 2G\Delta\epsilon'_{ij} + \sigma'_{ijo} \tag{27}$$

where G is the shear modulus,

$\Delta\epsilon'_{ij}$ is the deviator strain increment and

σ'_{ijo} is the deviator stress on the previous cycle.

For yield calculations, the effective stress from von Mises plasticity theory is used

$$\bar{\sigma} = \sqrt{\frac{3}{2} \sum_{i,j} \sigma'_{ij}\sigma'_{ij}} \ . \tag{28}$$

Yield occurs for $\bar{\sigma} \geq Y$, the yield strength.

A linear work-hardening is assumed such that

$$Y = Y_o + Y_D \bar{\epsilon}^p \tag{29}$$

where Y_D is the work-hardening modulus

and $\bar{\epsilon}^p = \int \sqrt{\frac{3}{2} \sum_{i,j} d\epsilon^p_{ij} d\epsilon^p_{ij}}$, the equivalent plastic strain, defined in terms of the plastic strain deviator increments $d\epsilon^p_{ij}$.

Strain-rate effects are treated with the linear viscous, one-parameter model

$$\frac{d\sigma'_{ij}}{dt} = 2G \ \frac{d\epsilon'_{ij}}{dt} - \frac{\sigma'_{ij} - Y \dfrac{d\epsilon^p_{ij}}{d\bar{\epsilon}^p}}{T} \tag{30}$$

where T is a time constant. This equation is presumed to govern only when the stresses exceed yield. When work-hardening is combined with

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the rate effects, the solution must be obtained numerically for general cases. The solution for stress derived by Seaman et al.[2] is

$$\sigma'_{ij} = \sigma'_{ijo} \exp[-(t - t_o)/T] \tag{31}$$

$$+ \left[\frac{Y_1 + \frac{Y_D \bar{\sigma}^{-N}}{4G}}{1 + \frac{Y_D}{4G}} \cdot \frac{\sigma'_{ij}}{\bar{\sigma}^{-N}} + \frac{T}{t - t_o} \left(\sigma'^{N}_{ij} - \sigma'_{ijo} \right) \right] \left\{ 1 - \exp[-(t - t_o)/T] \right\}$$

where $t - t_o$ is the duration of the time increment

Y_1 is the yield strength at the beginning of the increment

σ'^{N}_{ij} and $\bar{\sigma}^{-N}$ are elastically computed deviator stress and effective stress from Eqs. 27 and 28. The plastic deviator strain increments govern the nucleation and growth processes. These strains are computed as the difference between the total deviator strains and the elastic values.

B. <u>Simulations of Fragmenting Round Experiments</u>

Computer simulations were made for three fragmenting cylinders of HF-1 high fragmentation steel, one cylinder of Armco iron, and one HF-1 fragmenting round of standard munition geometry. The shear band sub-routine SHEAR2 was used as the stress-strain relation in all cases. Four of the cases were considered to be cylindrically symmetric, and there-fore calculations were made with the one-dimensional wave propagation code PUFF. Since the fifth round (of HF-1) had the shape of a standard munition, a two-dimensional wave propagation code, TROTT, was used for that case. Adjustments were made in the two codes so that the identical SHEAR2 subroutine was used with each. The dimensionality of the codes

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refers to the number of directions along which quantities can vary (radial in PUFF, radial and axial in TROTT). In both cases the stresses, strains, and damage have components in all three dimensions.

The geometries of the five experiments are shown in Figures 16 and 17. Three are configurations used by Crowe et al.;[4] the other two are part of this work. Experiment 5084-3 was simulated first with TROTT and SHEAR2 (with no damage permitted), and the pressure history at mid-length on the inner radius of the round was fitted to an exponential. This exponential pressure history was used for the SRI PUFF simulations of that experiment. For all other calculations, the explosive was simulated as a detonating material. In the cylindrically symmetric cases (SRI PUFF calculations), the explosion was treated as a simultaneous detonation. For the TROTT problem, (the fragmenting projectile) the explosive was simulated with a running detonation. All explosives were treated as polytropic gases except the PETN in 5084-4, which was handled with a tabular (pressure-volume) equation of state. The tabular values were obtained from a TIGER detonation calculation.[5] More details about these calculations, including the data decks for the calculations, are given in Appendix B.

The results of the simulation of 5084-3 were simply that the maximum plastic shear strain was 1.15 times the threshold value. Because the experiment showed only slight evidence of the beginning of shear banding, there seems to be a satisfactory correlation of test and calculation.

The fragment size distributions for the three cylinders that showed complete separation are shown in Figure 18. The computed results are also shown. The two tests of Crowe occurred at a strain rate of about 2×10^4/sec through the period of high damage while experiment 5084-4 occurred at a strain rate of 6×10^3/sec. The experimental results with HF-1 show that an increase in strain rate leads to a much larger

45
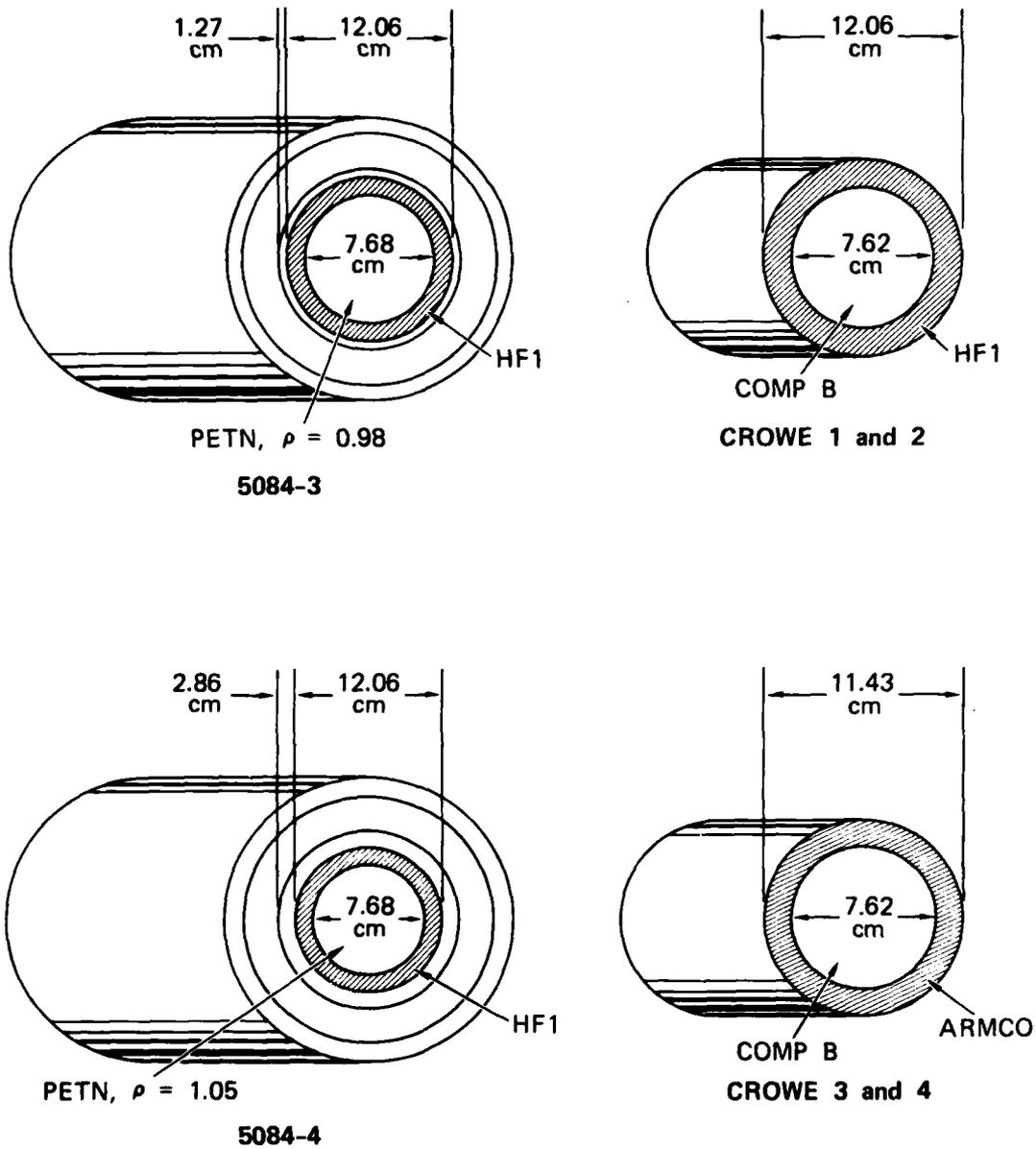
FIGURE 16   FOUR CYLINDRICAL FRAGMENTING ROUND CONFIGURATIONS TREATED
WITH SRI PUFF AND THE SHEAR BAND MODEL

DISCARDING ROTATING BAND

HF-1

JOINT IN CASE

SECONDARY EXPLOSIVE PBXN-106

PRIMARY, CH6

PLASTIC CONTAINER FOR EXPLOSIVE
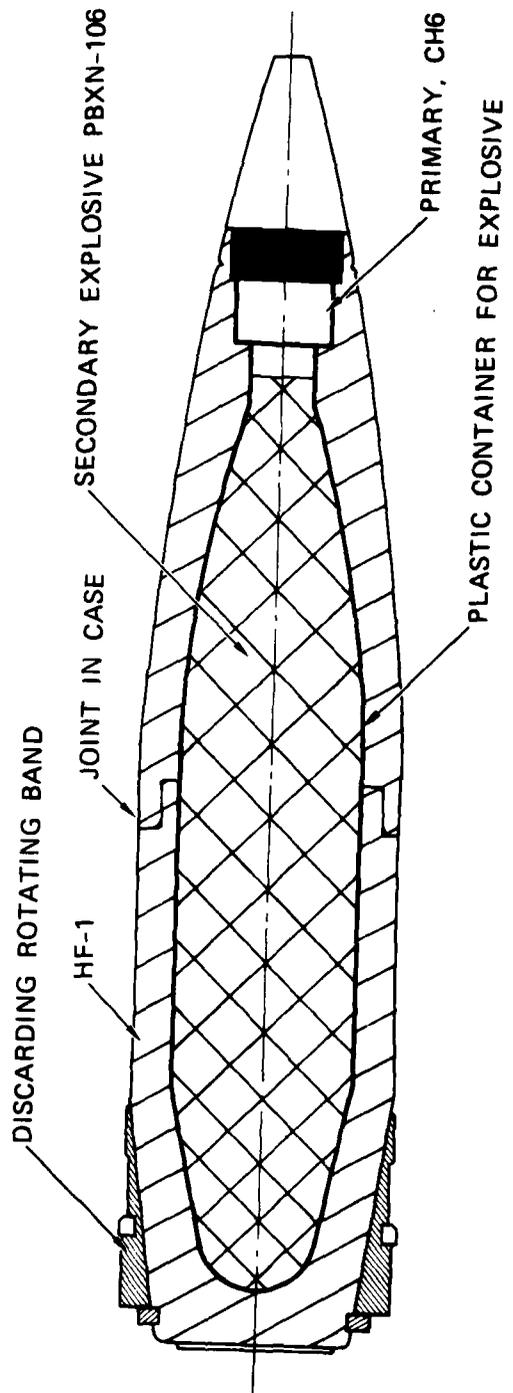
MA-5084-27

FIGURE 17   SCHEMATIC OF CROWE'S SAMPLE FRAGMENTING PROJECTILE

47
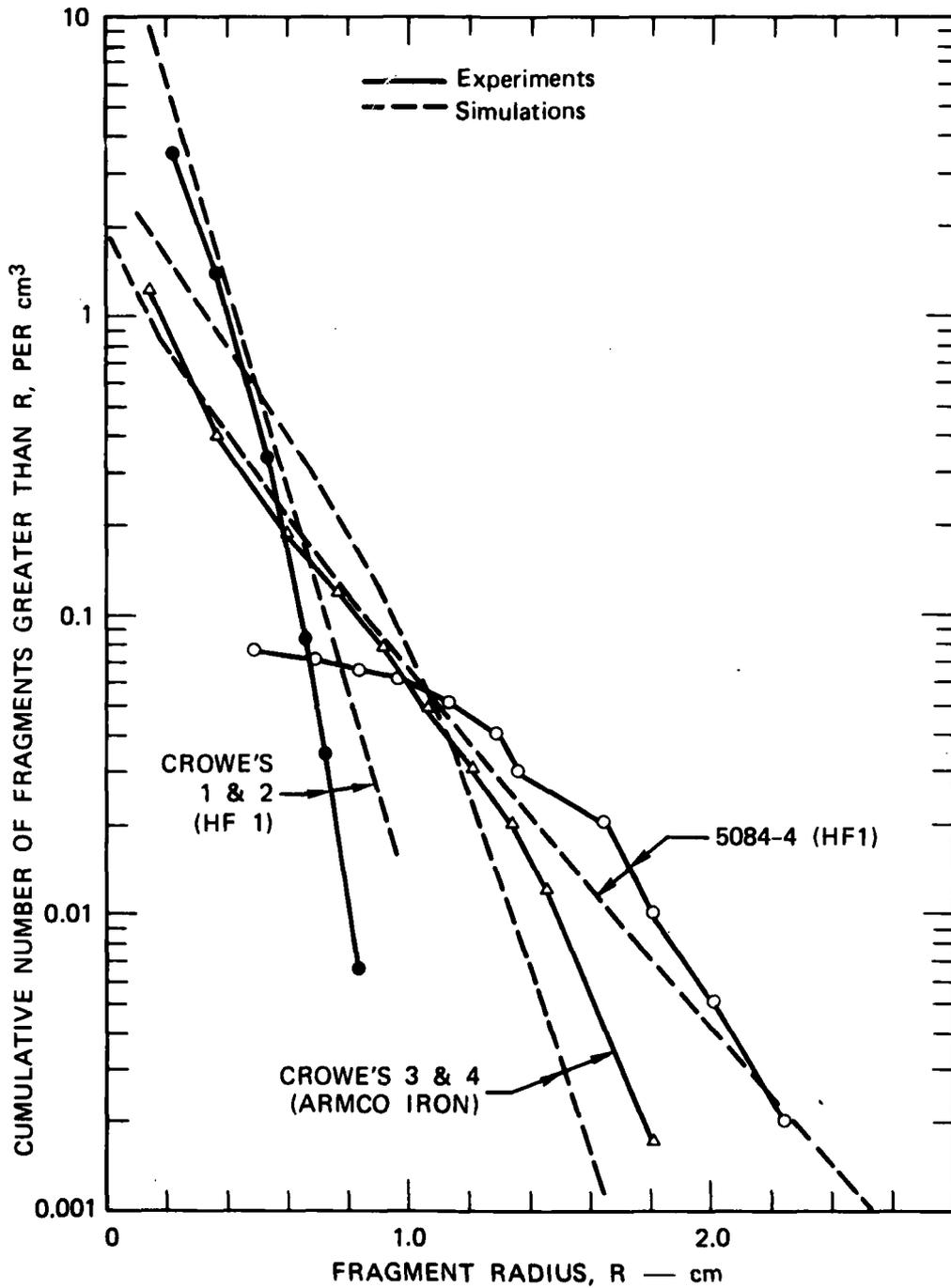
FIGURE 18   COMPARISON OF EXPERIMENTAL AND COMPUTED FRAGMENT
SIZE DISTRIBUTIONS FOR HF-1 AND ARMCO IRON CYLINDERS

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number of fragments, and consequently to smaller fragments. The Armco
iron test, while conducted at the high strain rate, shows a fragment
size distribution more like that from experiment 5084-4, which was
conducted at the lower strain rate. This difference may be associated
with the smaller number of inclusions in Armco iron, or with the lower
yield strength; the data base is not yet adequate to provide such an
association.

The marked difference in the distributions measured in the two
HF-1 experiments provide a good test of the model. The same parameters
were used in both HF-1 calculations. The correspondence between computed
and observed distributions indicates that the model can account for
the differences in strain rate, geometrical layout, and explosive
pressure. The Armco iron calculation was run with a nucleation co-
efficient one-third that for HF-1 ($C_n = 10^{-3}$ for HF-1, 3×10^{-4} for
Armco). This difference in parameters probably arises because of the
fewer nucleation sites in Armco iron. The nucleation plastic strain
threshold probably should also have been changed for Armco iron, but
no value was known.

The PUFF simulations of the three cylinder experiments showed
that the contained rounds required much longer to reach full fragmenta-
tion than the unconfined rounds. Crowe's experiments took about 45 μsec
to reach full fragmentation,[4] according to the calculations, while the
contained cylinder required over 200 μsec. In all three cases the
shear banding began at the inner radius and gradually moved out. All
computational cells showed shear banding in all three of the active
orientations illustrated in Figure 8. However, in each case, orienta-
tion 1 was the first one activated; the others began to nucleate bands
much later. When the bands from the three orientations were added,
orientations 2 and 3 contributed only a few percent at most. This
result agrees with the experiments, which indicate that most bands
are in the first orientation.

The two-dimensional simulation of Crowe's sample fragmenting projectile was performed with the cell layout shown in Figure 19. All the metal parts are of HF-1 steel. No provision was made for the joint in the two parts of the case. The primary explosive in the fuse is CH6 and the secondary explosive is PBXN 106. The simulation includes treatment of a running detonation that starts in the primary and sweeps at the C-J velocity through the secondary explosive. The progress of the running detonation is evident from the cell motion in Figure 20, which shows a series of plots from the two-dimensional calculations. The indicators in the figure show the attendant progress of fracturing in the HF-1 cells.

Figure 21 shows a comparison of the computed fragment size distribution with the measured distribution. The computed distribution was obtained by summing the fragment size distributions from all computational cells, accounting for the relative mass associated with each cell. Cells that did not fragment were mainly in the nose, and all unfragmented masses were added in as a single large chunk. In this two dimensional simulation, the first shear band mode activated was usually orientation 1 at 45° between radial and circumferential directions. Usually this mode also dominated at the time of fragmentation. Considerable shear strain also occurred along planes of constant radius or axial distance, and if bands had been permitted on these orientations, some shear bands would have occurred in the calculations. The duration of detonation was about 65 μsec. By 110 μsec, nearly all the shear banding cells were completely fragmented. No shear banding occurred forward of K-row 48.

Having completed these simulations of the fragmenting rounds, we then examined ancillary aspects of the computed results to see how these aspects matched the data. Among these aspects were the amount of homogeneous plastic strain, the effect of microcracking, the relative importance of the nucleation process, and the information that can be obtained from the shape of the fragment size distribution.

50
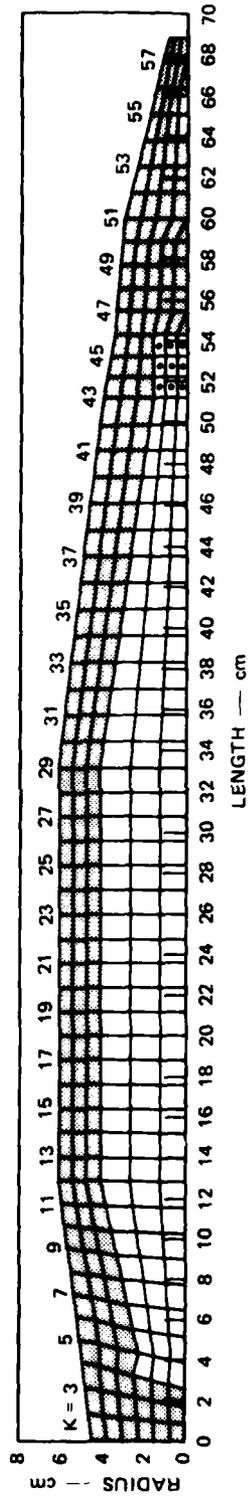
FIGURE 19   CELL LAYOUT FOR TWO-DIMENSIONAL SIMULATION OF CROWE'S SAMPLE FRAGMENTING PROJECTILE

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• Some Shear Banding
■ Fragmentation by Shear Banding

2.41 μsec

39.1 μsec

58.8 μsec

75.1 μsec

89.9 μsec

104 μsec

107 μsec

MA-5084-41
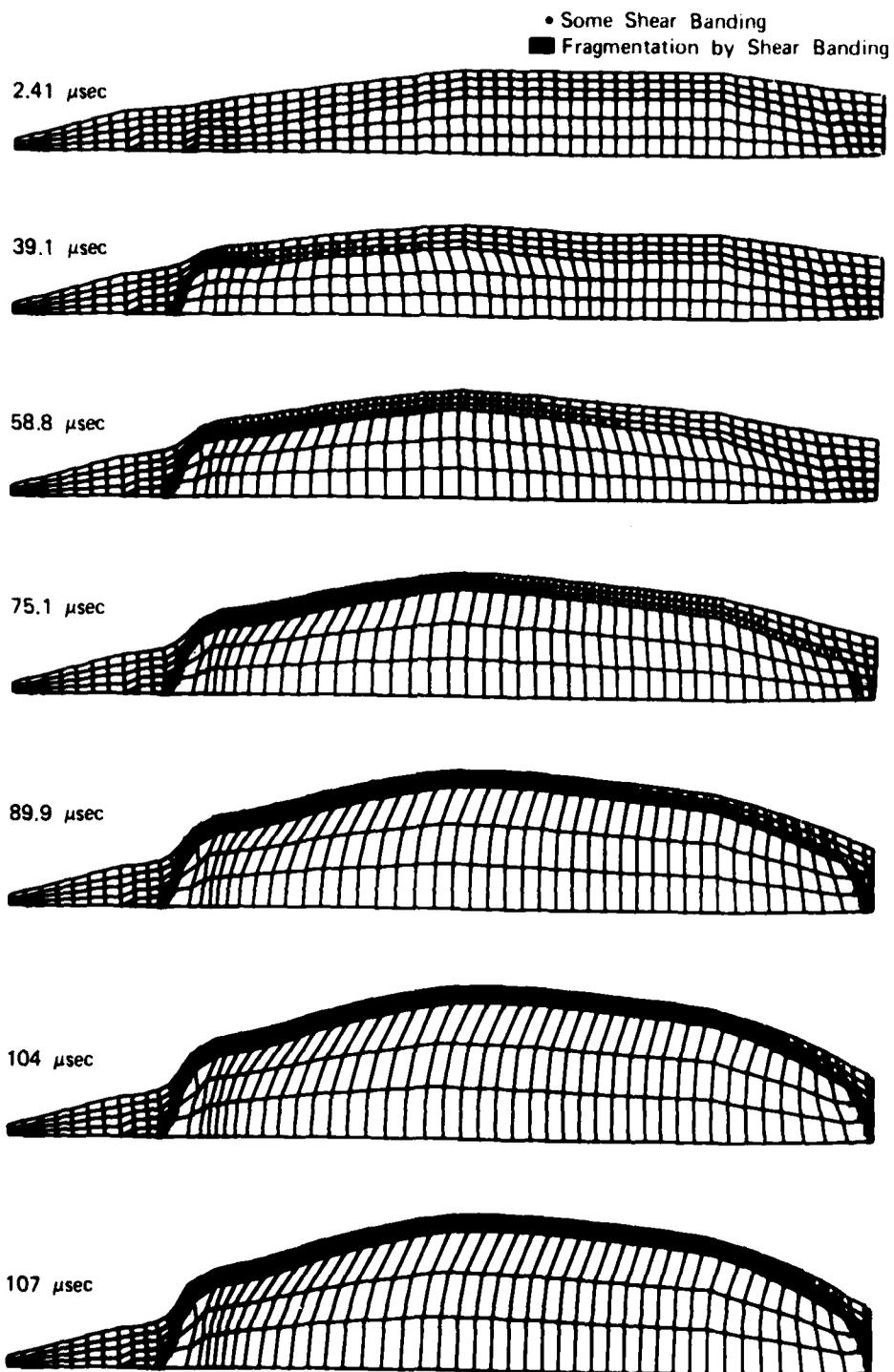
FIGURE 20   PROGRESS OF DETONATION AND SHEAR BAND DAMAGE
COMPUTED IN THE TWO-DIMENSIONAL SIMULATION
OF CROWE'S SAMPLE FRAGMENTING PROJECTILE

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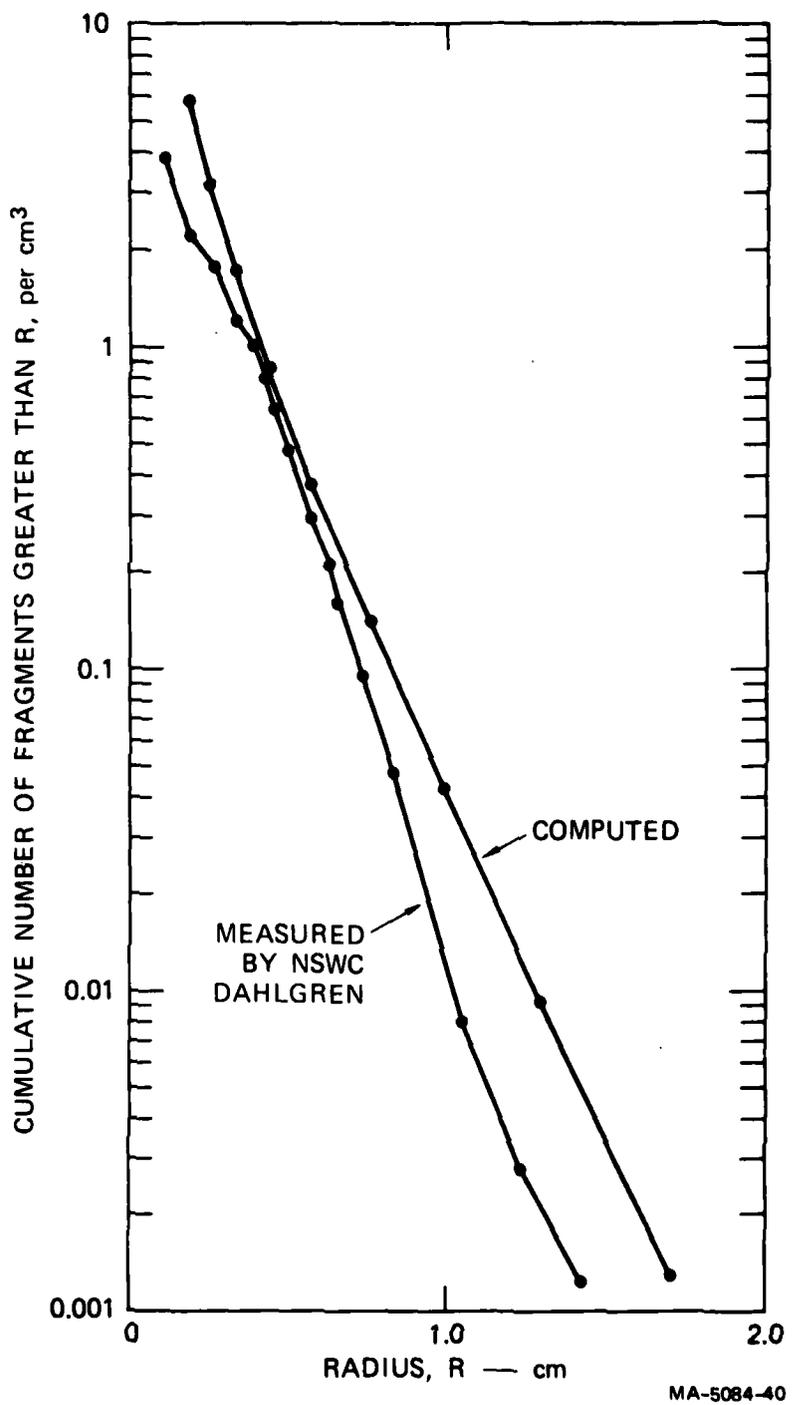
FIGURE 21   COMPARISON OF COMPUTED AND OBSERVED FRAGMENT SIZE
DISTRIBUTIONS FOR CROWE'S SAMPLE FRAGMENTING PROJECTILE

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In the model calculations, it was assumed that the shear band
nucleation and growth gradually takes a larger portion of the total
plastic strain until all is being absorbed by the bands. This means
that after the threshold of 30% equivalent plastic strain is reached,
the plastic strain taken in homogeneous deformation will continue
to increase until fragmentation occurs. In our computed results the
total equivalent plastic strain imposed is 60 to 80%. But the homogeneous
plastic strain at the time of fragmentation is about 40%--the rest is
used in producing bands. Experimental measurements indicate that there
was only 35% homogeneous strain in the fragments. It may be necessary
to reexamine the nucleation and growth processes to find means whereby
the band formation mechanism can absorb a larger fraction of the induced
plastic strain.

In the calculations, fragmentation was assumed to occur only by
shear banding, whereas in some of the experiments microcracking in the
outer layer participated in the fragmentation process. In future work
the effects of microcracking must be included in the computations.

The computed fragment size distributions were obtained by varying
mainly the nucleation rate coefficient. The Orowan equation appears
to force a balance between nucleation and growth so that an increase in
nucleation automatically results in less growth.

The fragment size distributions generally are linear on a semilog
graph of cumulative number versus fragment radius. However, there are
some interesting deviations from linearity in the calculations and the
experiments. A concave upward portion for small radii results from
the nucleation of more bands at a late stage. A concave downward
portion at the large radius end results from reaching a fixed velocity
limit such as the shear wave velocity.

C. Conclusions and Recommendations

A constitutive model was developed to provide elastic-plastic stress-strain relations and to represent the shear banding phenomena observed in a series of fragmentation experiments in HF-1 steel. The model includes the nucleation of shear bands, growth of bands, and coalescence of bands to form fragments. The model accounts for stress, strain, and damage quantities in three dimensions, although it is used in one- and two-dimensional wave propagation codes. The model permits bands to grow in six specific orientations; these include the three that can occur in exploding cylinders plus three more that may be activated in impacts. The developing damage causes relaxation of the stress in directions normal to the planes of the bands; hence, growth of bands develops an anisotropy in the material.

The results of calculations with the shear band model include stress histories, and the size and number of bands in each orientation. The band distributions can be transformed to a fragment size distribution and summed over all the material to form a Mott plot.

Five experiments were simulated: four of HF-1 steel and one of Armco iron. Three cylinders of HF-1 showed good correspondence between experimental and computed fragment size distributions although the strain rates, geometry, and explosive power were significantly different for the three cases. The shell in the shape of a standard round of HF-1 and the Armco iron cylinder also showed good correspondence between experimental and computed fragment size distributions. The model gave the correct orientation for most of the bands, and partitioned the plastic strain between homogeneous strain and that absorbed during shear banding. These results indicate that the nucleation and growth processes of the model are appropriate and are able to account for strain rate, geometry, and pressure differences.

The present shear band model was developed for the current experimental data on HF-1 steel, and has been verified only for those data. These data are for cylindrical geometry which is not sensitive to many features of the model: hence, those features are not constrained by the data. To develop a model representing a wide range of material types, loading conditions, and geometries will require more experiments and coordinated model development. Specific areas of the model that are recommended for further study are the following:

(1) The nucleation law requires verification experimentally, or modifications to match experiments. Associated experiments should be conducted for a broad range of strain rates and for various levels of damage. Especially important are experiments at the level of incipient damage, because at that point the nucleation process has been little affected by the development of damage.

(2) The growth process should be examined in the light of the experimental data obtained in (1) above. The low-damage experiments should guide selection of a new function, if necessary, and evaluation of the parameters. Band motion may also be examined analytically by studying the behavior of a single band in homogeneous material. Band width should be determined as a function of strain rate, thermal conductivity, thermal strength reduction, and work hardening. These width calculations can probably be performed with a one-dimensional shear wave propagation code. Growth of the bands may be treated in the same way with a two-dimensional code. If both width and growth velocity are governed by homogeneous processes, these computational studies could aid greatly in defining some of the micro-mechanisms of the shear band process.

(3) In impacts and fragmenting rounds, brittle fracture and shear
     bands combine to form fragments, and this combined process is
     not yet incorporated into the NAG (Nucleation and Growth)
     models.* For this process, fragments as well as material at
     incipient damage levels should be examined. Then analytical
     techniques for joining the two damage processes should be
     developed.

(4) In the model, both nucleation and growth in any direction are
     presumed proportional to the plastic shear strain in the rele-
     vant direction. This hypothesized relationship should be
     examined in the light of data from incipient damage experiments.

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*The combined process was accounted for in the work reported in Ref. 2
by permitting cells to undergo either brittle fracture or shear banding,
but not both. In that study only one orientation of shear bands was
permitted.

V OBLIQUE ROD IMPACT - EXPERIMENTAL PHASE

The mechanisms for fracture of a long-rod penetrator under high velocity impact (either oblique or perpendicular) with an armor plate are inadequately understood. Large pressures and large shear deformations at high strain rates make this a difficult problem to analyze or model.

In a preliminary effort to establish the key penetrator failure modes, we performed the following tasks:

(1) Examined metallographically the distal ends of steel and tungsten alloy long-rod penetrators recovered from experiments at Lawrence Livermore Laboratories (LLL).

(2) Examined metallographically fragments that LLL recovered from the proximal end of obliquely-impacted tungsten alloy rods.

(3) Performed a series of oblique-angle long-rod penetrator experiments using HF-1 steel to study break-up characteristics as a function of velocity.

(4) Examined the fragments recovered from a long-rod tungsten alloy penetrator perpendicular impact performed at the Ballistic Research Laboratory.

These activities are described in detail in the following pages.

A. LLL Experiments

From Dr. John Scudder and Mr. Leroy Hoard of LLL, we obtained fragments recovered from scale-model long-rod penetrator oblique-impact tests performed in their laboratories. The projectiles all had a diameter of 3.9 mm, a length of 39 mm, and a hemispherical nose, and were impacted at a velocity of $\approx$ 1 km/sec and a 55° angle of obliquity into a target of mild steel with an overlay of AD85[*] alumina ceramic.

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[*] Product of Coors Porcelain Co., Golden, Colorado.

We first examined the distal halves of the projectiles---the only
fragments large enough to be recovered without any special recovery tech-
niques. Figure 22(a) shows a metallographic specimen made by cutting
through the axis of the specimen parallel to the plane of greatest
bending (all subsequent specimens were cut in the same way). The material
is Saratoga AISI-01 oil hardened tool steel ($R_c \approx 48$). Photomicrographs
of a region near the forward edge of the specimen show clearly a shear
band intersecting the edge and a tensile crack extending part way down the
length of the shear band. Since this was the only internal crack visible
in this specimen and since there was also a transformed region (indicating
a shear band) along the fractured forward edge of the specimen, there is
evidence that shear banding and subsequent tensile failure along a shear
band is an important failure mechanism for a steel long-rod penetrator,
at least in the distal regions.

We next examined the distal halves of tungsten alloy projectiles.
The alloy contained 90% tungsten, 7% nickel, and 3% iron by weight, and
was composed of nearly spherical tungsten grains $\approx 5 \mu$m in diameter in
a matrix. Unlike steel, tungsten does not exhibit a readily observed
phase change under high temperatures, which can be used as a clear
indicator of shear banding. However, this tungsten alloy does have an
advantage over some steel in that strain deformation is indicated by grain
elongation. Figure 23 shows three photomicrographs from a tungsten
penetrator---one taken from an undeformed region, showing largely circular
grain cross sections; one from a region of large but primarily homogeneous
shear deformation, showing grain cross sections with aspect ratios of
around 4 to 1; and one from a region along the proximal edge where
inhomogeneous shear deformations have elongated grains to aspect ratios
as high as 20 to 1, and caused shear cracking.
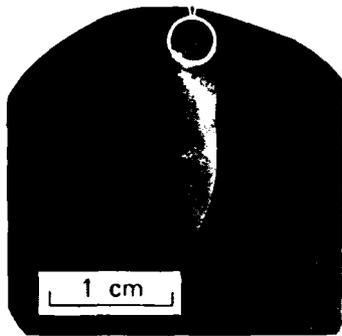
(a)
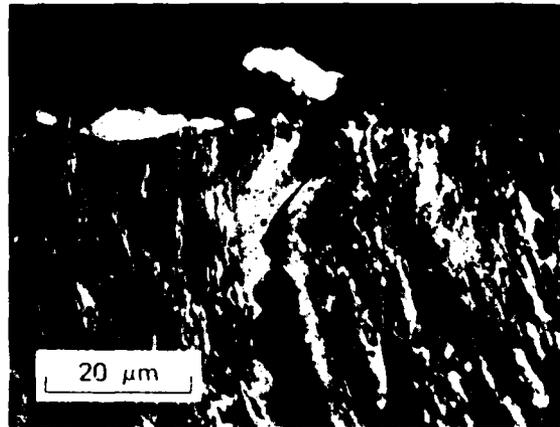
(b)
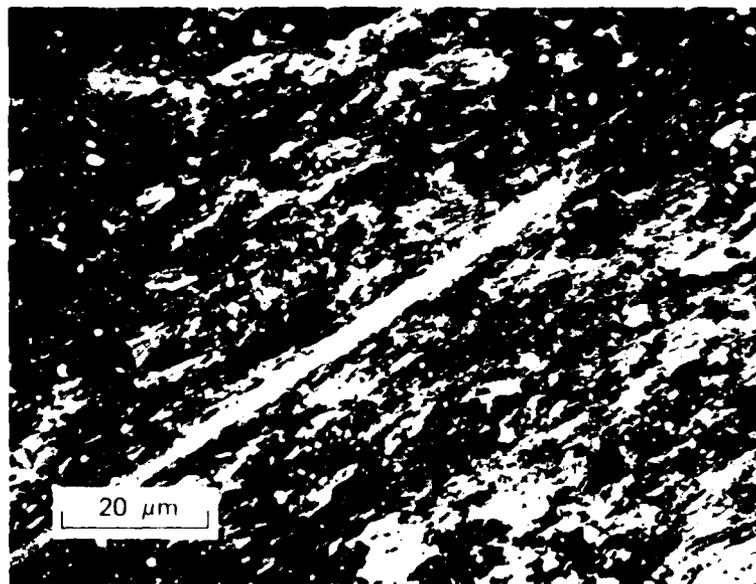
(c)

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FIGURE 22   SPECIMEN (a) FROM DISTAL END OF OBLIQUELY IMPACTED TOOL STEEL
LONG-ROD PENETRATOR FIRED AT LAWRENCE LIVERMORE LABORATORY
WITH PHOTOMICROGRAPHS (b) AND (c) SHOWING SHEAR BANDING

61
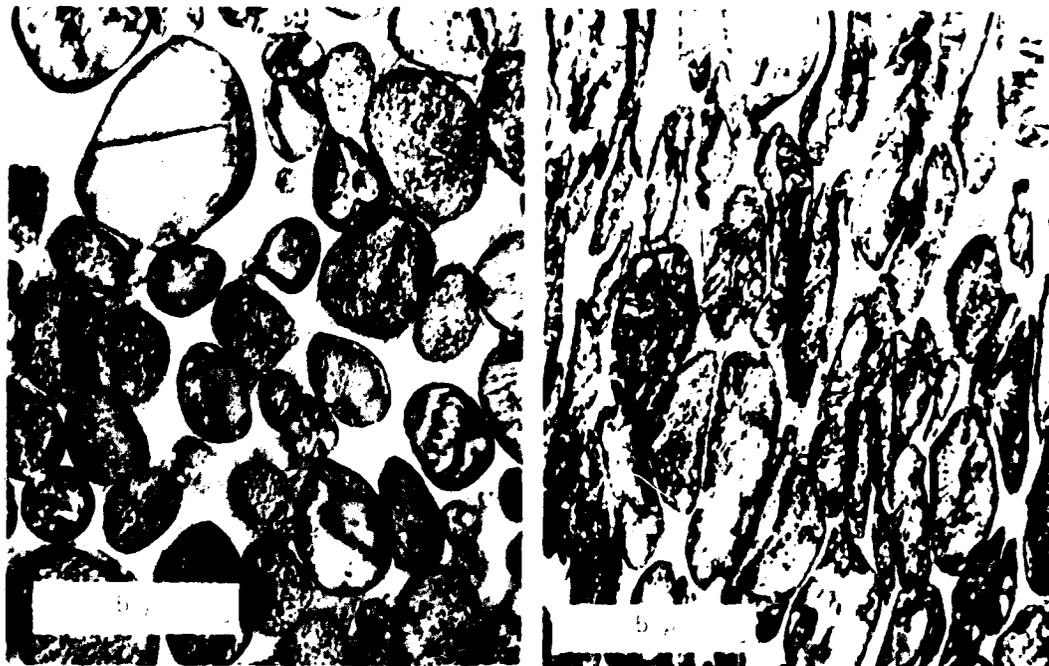
(a)

(b)
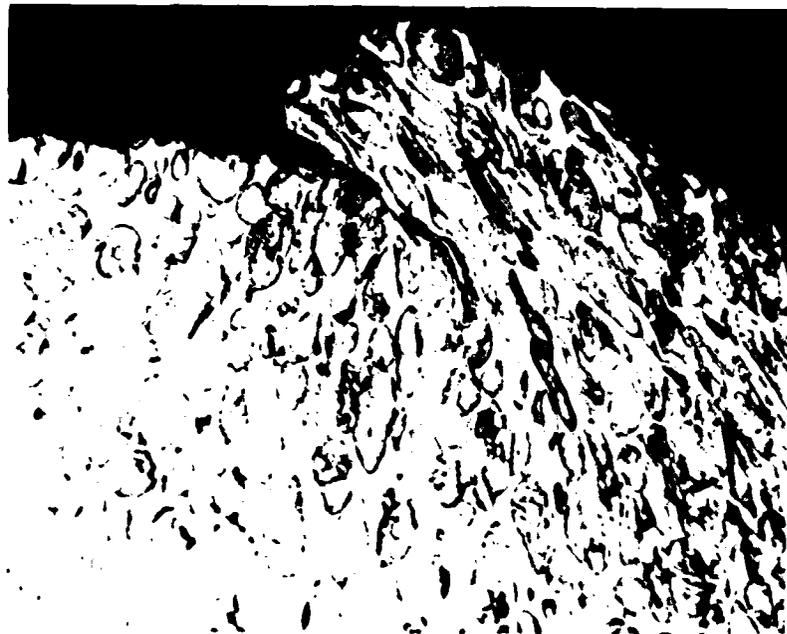
(c)

MP-5084-19

FIGURE 23   PHOTOMICROGRAPHS OF DISTAL HALF OF OBLIQUELY IMPACTED TUNGSTEN
ALLOY LONG-ROD PENETRATOR SHOWING (a) UNDEFORMED REGION,
(b) REGION OF PRIMARILY HOMOGENEOUS SHEAR DEFORMATION, AND
(c) REGION OF LARGE INHOMOGENEOUS SHEAR DEFORMATION RESULTING
IN SHEAR CRACK

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We have thus seen evidence of failure due to shear banding or inhomogeneous shear deformations in the distal region of both steel and tungsten alloy long-rod penetrators. Since the shear strains and strain rates would be higher near the proximal ends, this type of failure mechanism should be even more important there. To test this belief, we suggested to Dr. Scudder and Mr. Hoard a means of recovering fragments from the proximal halves of obliquely impacted long-rod penetrators by capturing them in a relatively soft rubber-like substance. They subsequently performed such experiments, and we examined several recovered tungsten alloy specimens one of which is shown in Figure 24. The structure is very similar to that shown in Figure 23. In Figure 24(b), a gradual transition from no shear deformations to large shear deformations takes place across the width of the specimen. And in Figure 24(c), narrow bands of extremely high shear deformation are seen between regions of moderate shear deformation. We cannot know precisely where in the penetrator this fragment originated, but it was within the proximal half.

Our conclusion is that failure by shear banding or inhomogeneous shear deformation occurs over a large region in a long-rod penetrator.

B. SRI Experiments

SRI's smooth-bore-gun facility, which had been used in previous armor penetration experiments,[1] was modified for use in scale-model long-rod penetrator oblique impact experiments. Hemispherical-nosed HF-1 steel projectiles 3.175 mm in diameter and 38.1 mm long were fired at a 55° angle of obliquity into 1-cm-thick steel armor plates. The relatively thick armor was chosen so as to minimize damage to the target, and thereby make the projectile failure easier to model computationally. HF-1 steel was chosen because it is a material for which dynamic fracture parameters are known.

(a)
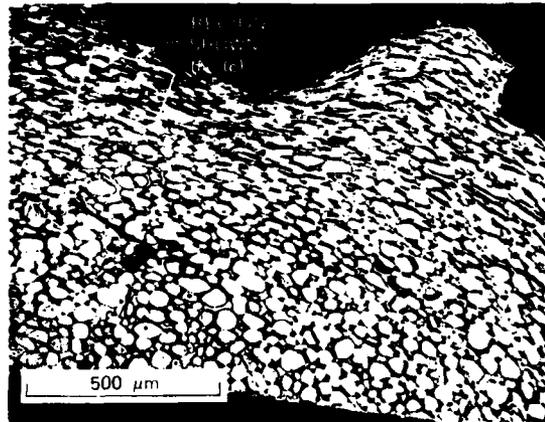
(b)
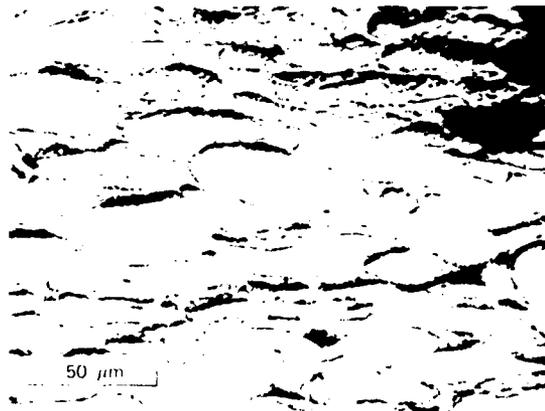
(c)

MP-5084-17

FIGURE 24   FRAGMENT (a) RECOVERED FROM OBLIQUELY IMPACTED TUNGSTEN ALLOY
LONG-ROD PENETRATOR FIRED AT LAWRENCE LIVERMORE LABORATORY
WITH PHOTOMICROGRAPHS SHOWING EVIDENCE OF LARGE HOMOGENEOUS
(b) AND NONHOMOGENEOUS (c) SHEAR DEFORMATIONS

We fired a series of shots at impact velocities ranging from 0.17 to 0.97 km/sec, and the results are shown in Figure 25. As the impact velocity increases to about 2/3 km/sec, the projectiles become more and more bent, eventually separating into several pieces. However, at higher velocities. the bending decreases and the projectiles undergo large cross-sectional deformations in their proximal regions. In all cases, target damage was negligible.

The damaged projectiles were sliced in half along their plane of maximum bending, polished, etched, and examined metallographically. The surfaces of separation appeared to result from brittle, or tensile failure, but there was no evidence of either shear banding or tensile failure in the proximal region. The higher velocity shots exhibited a transformed region along the outer, flattened surface, which undoubtedly resulted from the projectile sliding along the surface of the target as it was being deflected.

C.   BRL Experiments

Late in the program Dr. Zukas of Ballistics Research Laboratory sent us material recovered from a long-rod penetrator impact experiment performed at BRL. The material was the same tungsten alloy used in the LLL penetrators; however, the scale was larger--the hemispherical-nosed projectile was 0.79 cm in diameter and 7.9 cm in length, and the impact occurred at 0.850 km/sec with a 0° angle of obliquity into a 2.54-cm-thick steel target.

The recovered fragments included the distal quarter and an assortment of small fragments. Figure 26 shows a slice through the large fragment, with a close-up of the area that contains what appears to be a long partially separated shear band. Detailed photomicrographs, seen in Figure 27, prove that it is indeed a shear band. The high magnification insets clearly show bands less than 10 µm wide, across which the two opposite surfaces

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55° IMPACT   HF-1 PROJECTILES

0.97 mm/μsec

0.84 mm/μsec

0.61 mm/μsec

0.55 mm/μsec

0.50 mm/μsec

0.30 mm/μsec

0.17 mm/μsec

VIRGIN
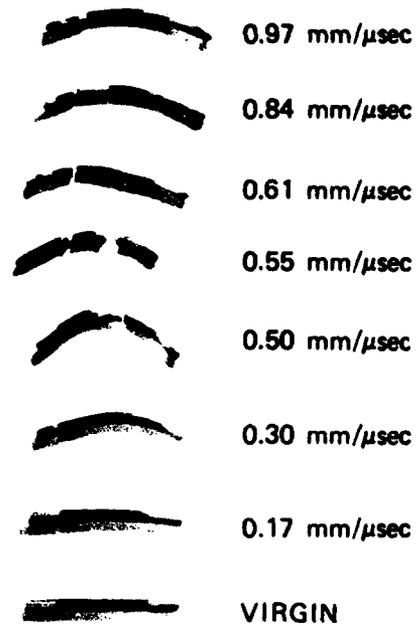
MP-5064-18

FIGURE 25   HF-1 STEEL LONG-ROD PROJECTILES
RECOVERED FROM OBLIQUE IMPACT
TESTS AT SRI INTERNATIONAL
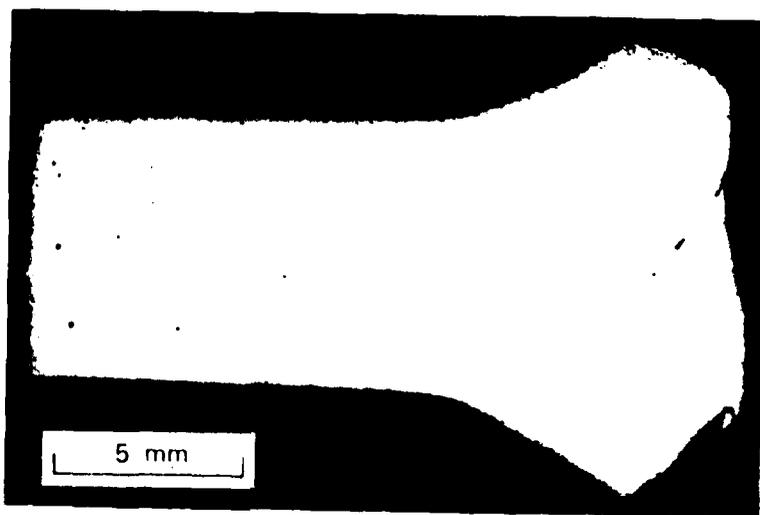
(a)

(b)

MP-5084-24

FIGURE 26   POLISHED SECTION (a) THROUGH BRL TUNGSTEN ALLOY
LONG-ROD PENETRATOR, WITH CLOSE-UP (b) OF SHEAR
BAND

67
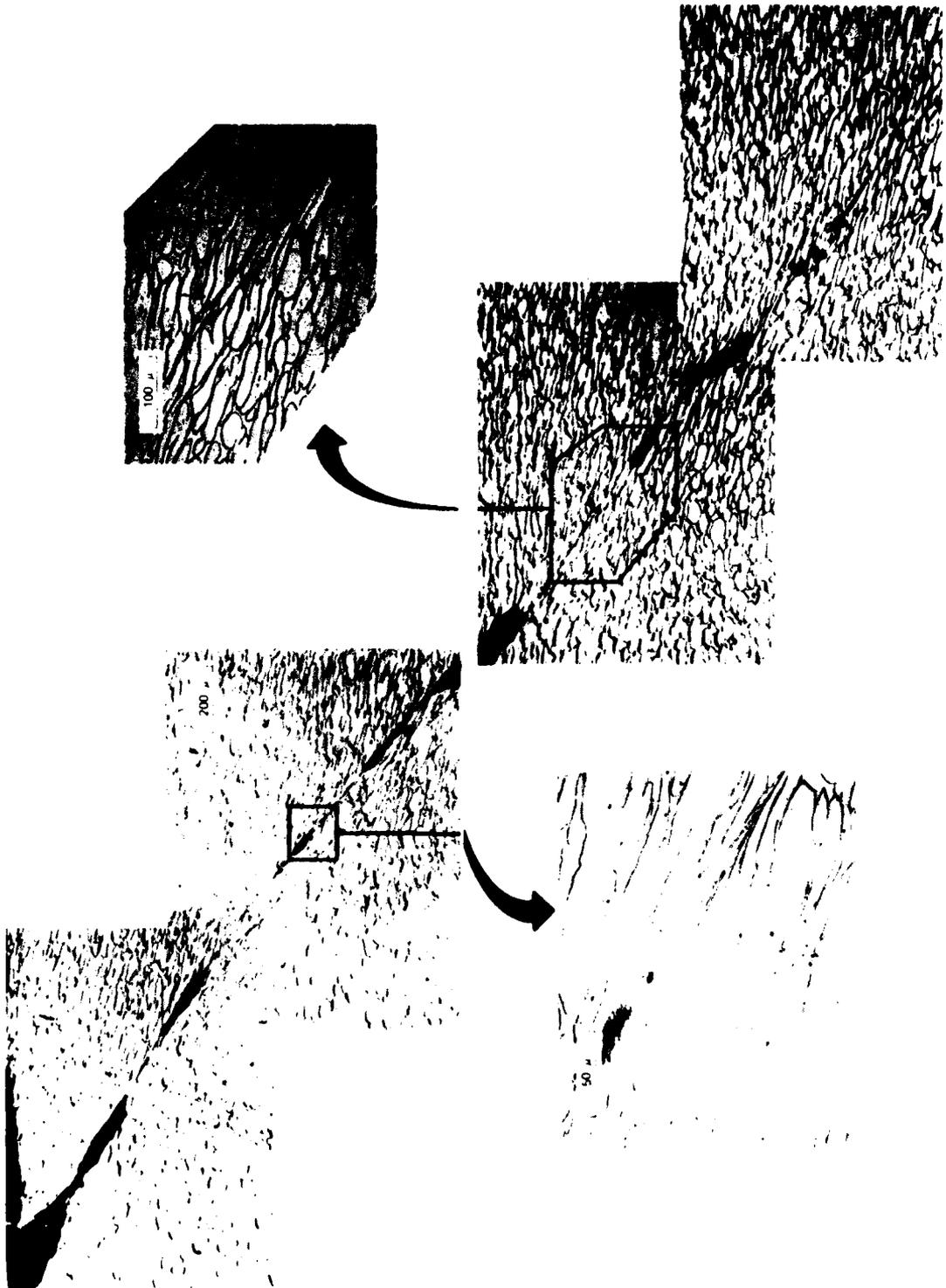
FIGURE 27   DETAILED PHOTOMICROGRAPHS OF SHEAR BAND IN BRL
TUNGSTEN ALLOY LONG-ROD PENETRATOR

have slid as much as 50 μm or more. The inset at lower left shows two such bands running parallel to each other about 60 μm apart.

A polished section through one of the smaller fragments is seen in Figure 28, along with two high magnification insets revealing shear-banded regions. These particular bands are quite curved, which indicates either that the material was in a highly nonuniform state of shear strain prior to the banding or that the material continued to undergo large homogeneous shear deformations after the band formed. Also, the highly deformed grain that exists away from the banded regions is indicative of the toughness of this alloy, which will sustain large amounts of shear strain before banding.

D. Conclusions

Shear banding is a principal mode of material failure in high-velocity impacts of tungsten alloy and steel long-rod penetrators, although the SRI experiments indicate that tensile fracture may also be important, particularly at larger impact angles.
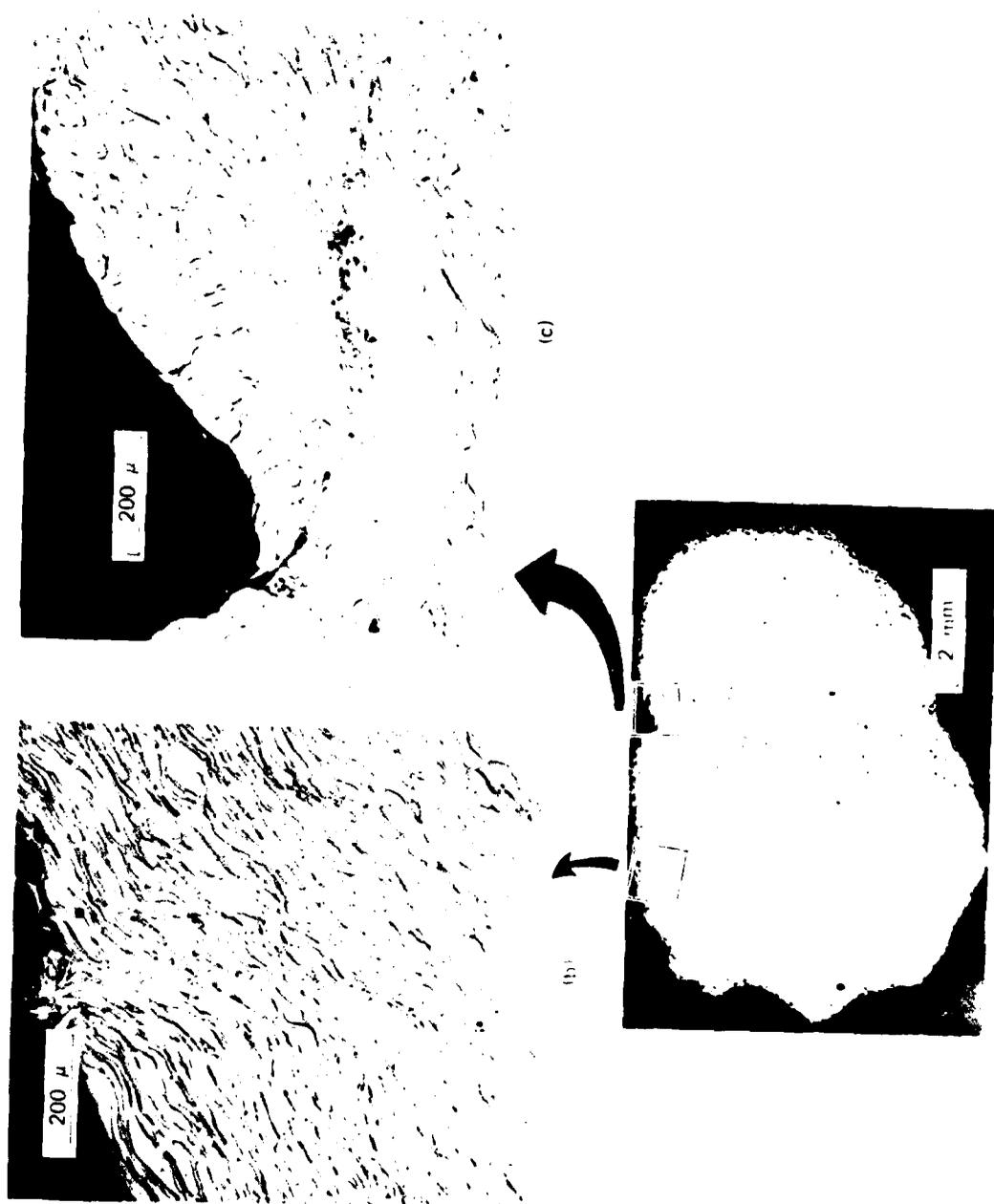
FIGURE 28   POLISHED SECTION (a) THROUGH FRAGMENT FROM BRL TUNGSTEN ALLOY LONG-ROD
PENETRATOR IMPACT, WITH CLOSE-UPS (b) AND (c) OF SHEAR-BANDED REGIONS

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VI OBLIQUE ROD IMPACT-COMPUTATIONAL PHASE

Oblique impact calculations were conducted with a two-dimensional wave propagation computer program. The calculations simulate a plate impacting on edge another plate at an oblique angle. Both the conditions in which the projectile sticks to the target and in which it slips were treated. For each condition, impact angles of 30°, 45°, and 55° to normal were considered.

The purpose of the calculations was to examine the phenomena associated with oblique impacts to gain insight into the reasons for breakup of the projectile and the mechanism of damage in the impact region. We examined the regions of fracture damage and locations near the point of impact where shear, compression, and bending stresses occurred, and we noted how these locations changed with the degree of slip along the target. We used an active fracture damage model (with stress-relaxation associated with damage) to see whether the damage would significantly affect subsequent bending and projectile breakup.

The computer program used is called TROTT. It is a two-dimensional, finite-difference Lagrangian code, and is similar to HEMP and TOODY. Three features made it attractive for this problem:

(1) TROTT is small; all storage is in core.

(2) The array storage is arranged to permit very large data storage for cells undergoing fracture.

(3) The code is simple and easy to modify for the special impact boundary conditions.

During the project, modifications were made to allow improved treatment of the impact problem.

The remainder of this section treats the special modifications made to the code TROTT, the geometries used in the calculations, and the results of the simulations.

A. Code Modifications

The TROTT code was altered to include a special nonnormal impact condition and triangular cells. The projectile is divided into the usual discrete finite-difference cells, whereas the target is treated as rigid and immovable. The rigid target was used for simplicity because the main interest was in breakup of the projectile at some distance back from the impact plane. It was felt that the conditions of sticking and slipping would bracket the boundary conditions actually experienced by a projectile.

For the slipping boundary condition, coordinate motion was permitted as usual until the point reached the target. Thereafter only tangential motion along the target face was allowed. For example, in Figure 29, Point A moves until it strikes the target at B and then slips to point D. Let (x_1, y_1) represent one point on the target. Then the equation of the target is

$$x = x_1 + (y - y_1) \tan \theta \qquad (33)$$

where θ is the angle of the target measured so that $\theta = 0$ is a normal impact. Then if a coordinate with the initial position (x_A, y_A), that is point A in Figure 29, moves with velocities $\dot{x}$, $\dot{y}$, impact occurs at point B at a time

$$\delta t = \frac{x_1 - x_A + (y_A - y_1) \tan \theta}{\dot{x} - \dot{y} \tan \theta} \qquad (34)$$

If δt is less than the current time step in the wave propagation calculations (i.e., point C is beyond point B), then the impact coordinate locations are at B

PROJECTILE

D

B

A

C

IMMOVABLE WALL

θ
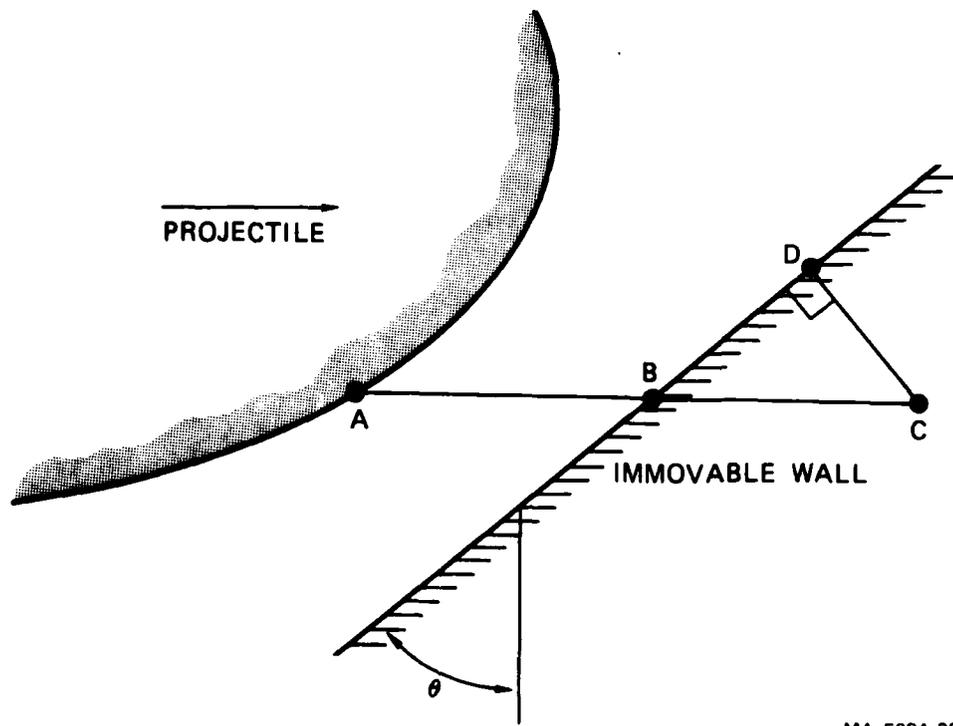
MA-5084-30

FIGURE 29   SLIDING OR STICKING CONDITIONS AT OBLIQUE TARGET

$$x_B = x_A + \dot{x}\ \delta t$$

<div align="right">(35)</div>

$$y_B = y_A + \dot{y}\ \delta t$$

After the impact, cell motion is tangential. The tangential velocity is

$$V_T = \dot{x}\ \sin\theta + \dot{y}\ \cos\theta\ .$$

The x and y coordinates of this velocity are

$$V_x = V_T\ \sin\theta$$

$$V_y = T_T\ \cos\theta.$$

The final coordinate locations are then

$$x_D = x_B + V_x\ (\Delta t - \delta t)$$

$$y_D = y_B + V_y\ (\Delta t - \delta t)$$

where Δt is the current time step.

The sticking boundary condition required that no motion occurs after impact. First, the coordinate motions were computed in the usual way. If a point had penetrated the target plane (i.e., point C in Figure 29), it was repositioned at the point where it first touched the target, point B, using Equation 35. Velocities of the impacting coordinates were set to zero.

A triangular cell feature was added to TROTT to handle the large distortion problem near the plane of impact. The usual quadratic cells become excessively distorted in the large shear flows near the front and they either halt the calculation or require rezoning of the cells.

The triangular cells are very resistant to cell distortion and therefore can follow the impact somewhat better. The triangular cells were constructed by simply dividing quadrilateral cells in two, and computing strain and stress separately for each half. The analysis followed the lines suggested by Johnson[6] for impact calculations.

Following incorporation of the triangular cells, we performed several short calculations with only a few cells to examine the new properties. The study, though not exhaustive, was adequate to guide our use of the triangular cells and to interpret the results. It revealed the following:

- The triangle Q artificial viscosity stresses used with quadrilateral cells are also needed with triangular cells to reduce oscillations.

- The time steps required for stability of triangular cell calculations are the same as those of the quadrilateral cell:

$$\Delta t = 0.9 \frac{\Delta X}{C}$$

  where ΔX is the length of the shortest side, C is the sound speed augmented to account for artificial viscosity, and 0.9 is the safety factor.

- In pure compression a pair of triangular cells each give stresses identical to the quadrilateral cell from which they were made.

- In shear, one of the triangular cells in a pair goes into tension, the other into compression, with the average of the two stresses being about that of a single quadrilateral cell. These stresses are often quite large, thus giving a misleading picture of the stress state.

The last item above shows that only simple equations of state should be used for the triangular cells. More complex constitutive relations with threshold levels and extreme nonlinearities would only give misleading results. Hence, in our impact calculations we used the triangular cells only for the first few rows of cells at the head of the projectile and permitted only elastic-plastic behavior in them.

B. Simulations

The geometry used in the calculations is shown in Figure 30. Two-dimensional plane strain conditions are used. The projectile slab has a rounded nose as shown so that one side of one cell is in continuous contact with the target at impact. A projectile velocity of 1 mm/μsec was used in all cases. The projectile slab, which is Armco iron, is treated as an elastic-plastic material in compression and with brittle fracture (BFRACT subroutine) in tension. In addition to the usual stress and motion results, moments and forces in the impacting slab were computed.

The resulting shapes of some of the projectiles after 14 μsec is shown in Figures 31 to 36.

Following impact, the projectile is gradually deformed so that it is in contact with the target over the entire projectile nose and for some distance along the side nearest the target. Projectile deformations appear as high shears that distort the impact-plane cells and elongate the cells just behind the impact. The cells are elongated through the thickness, suggesting a tendency toward splitting. Cells on the side of the projectile farthest from the target are also elongated in the axial direction, indicating a tendency to break in bending.

In all cases, the projectile tends to enlarge radially. This expansion is more evident for the shallower angles of impact. Associated with this is fracture in the radial direction by cells near the center of the projectile and axial fracture on the projectile surface farthest from impact.

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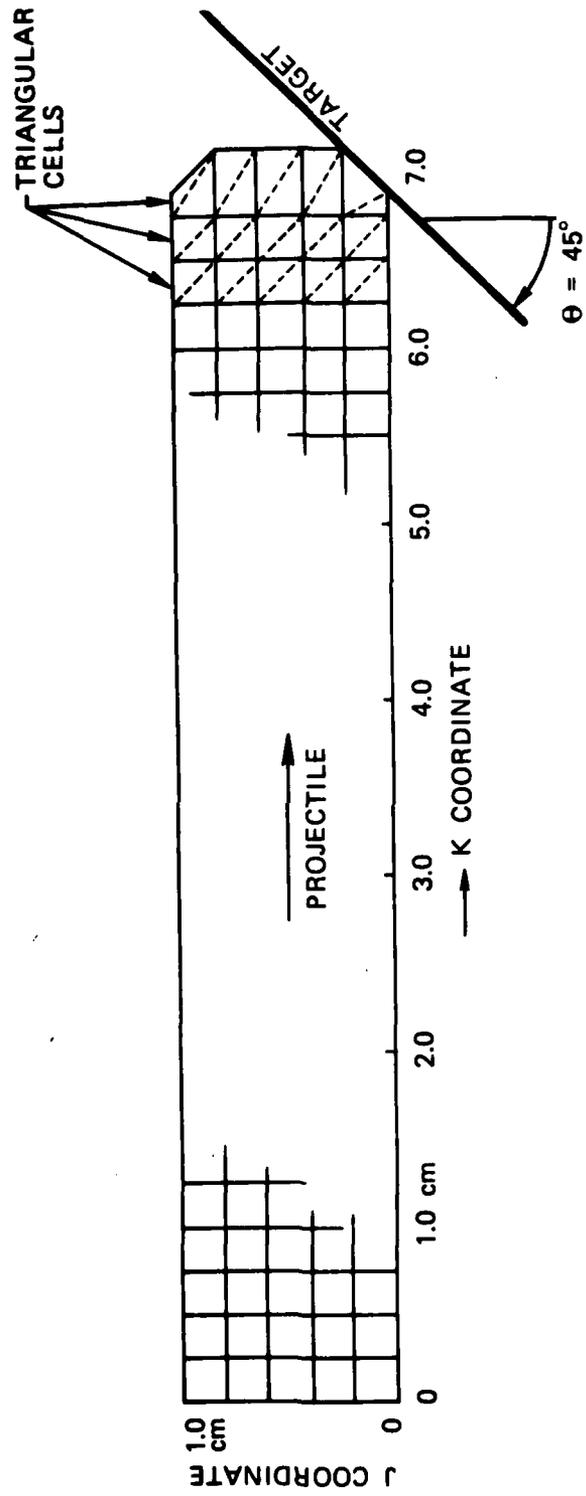
FIGURE 30   GEOMETRY FOR SIMULATIONS OF PROJECTILE PLATE IMPACTING A WALL
AT AN OBLIQUE ANGLE

77
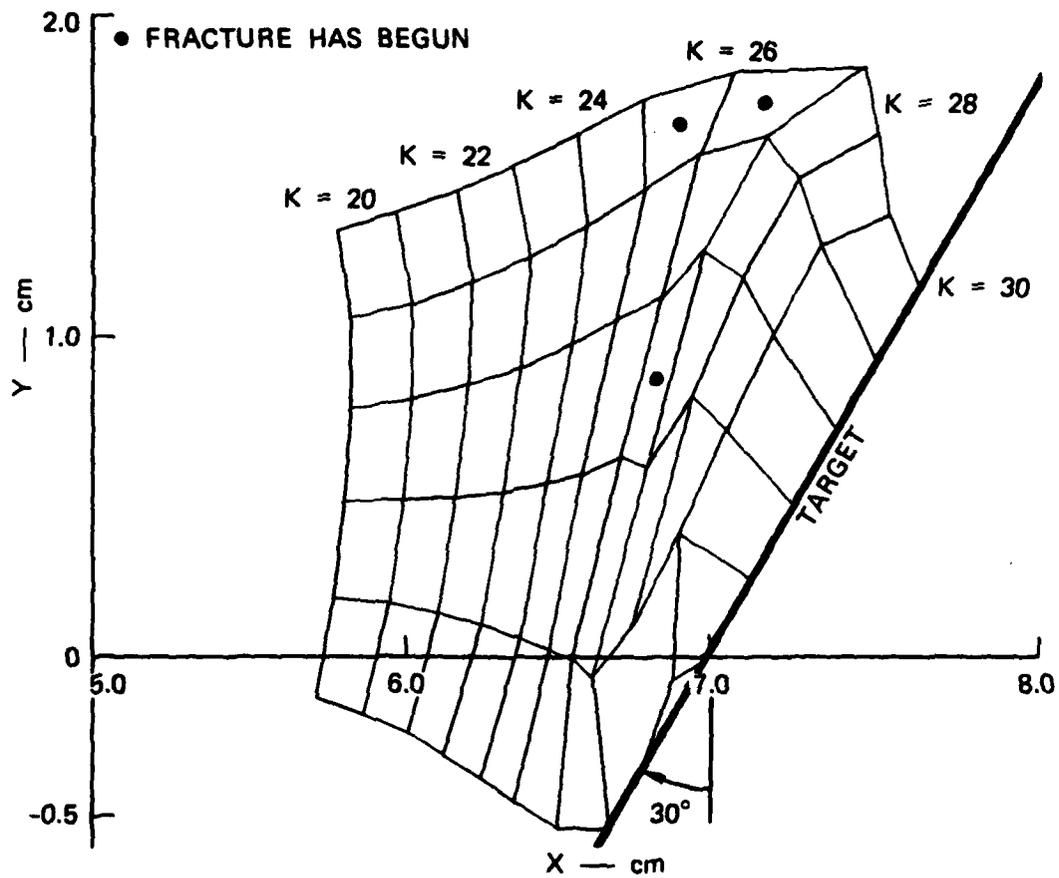
FIGURE 31   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 30°
OBLIQUITY, NO SLIPPING

MA-5084-32

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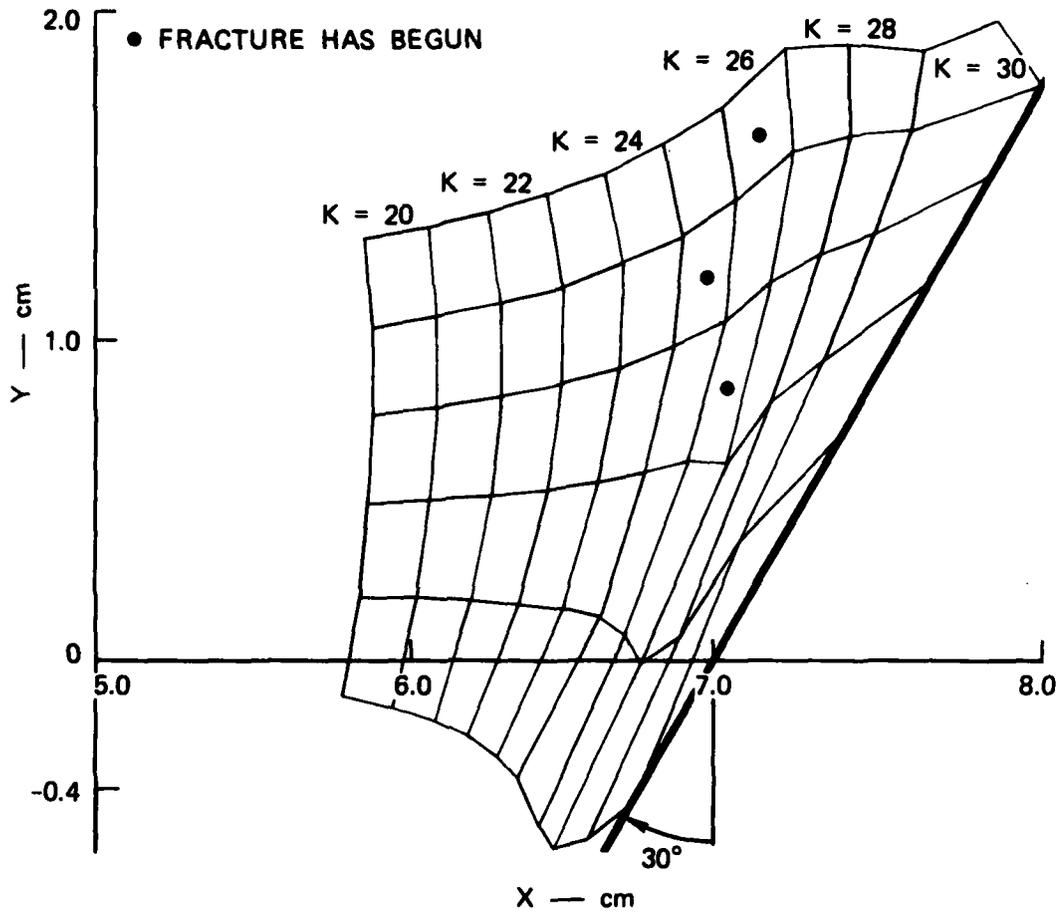
FIGURE 32   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 30°
             OBLIQUITY, SLIPPING

79
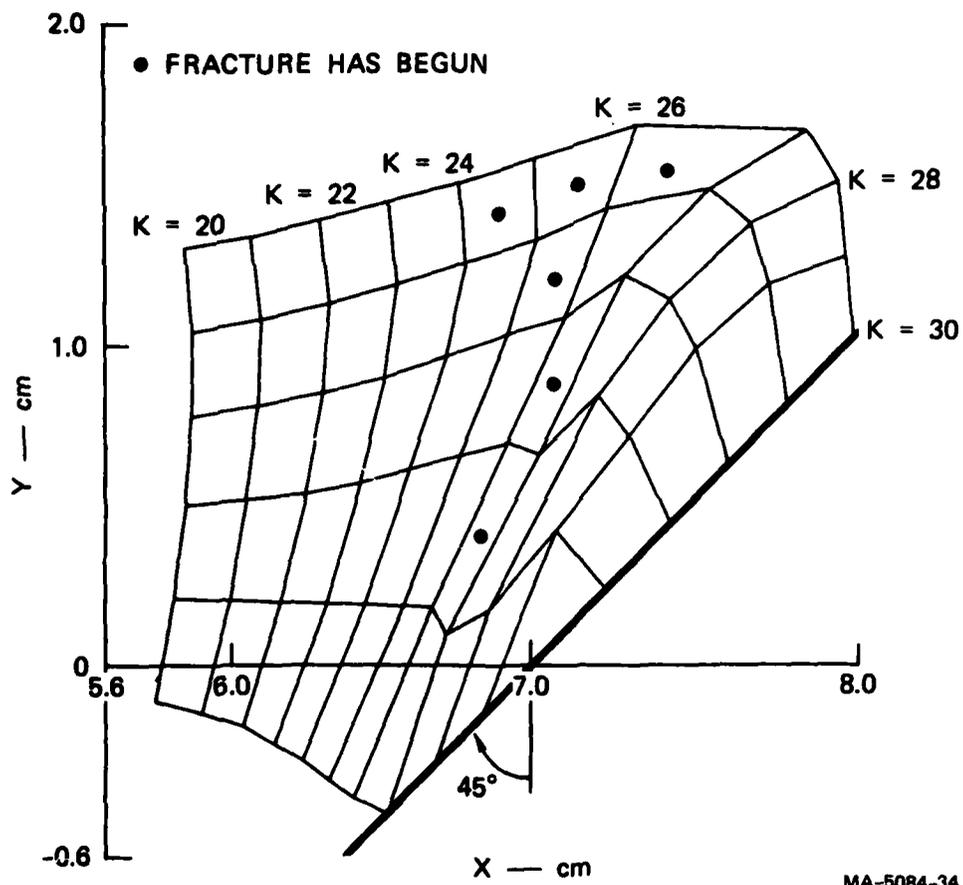
FIGURE 33   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 45°
OBLIQUITY, NO SLIPPING
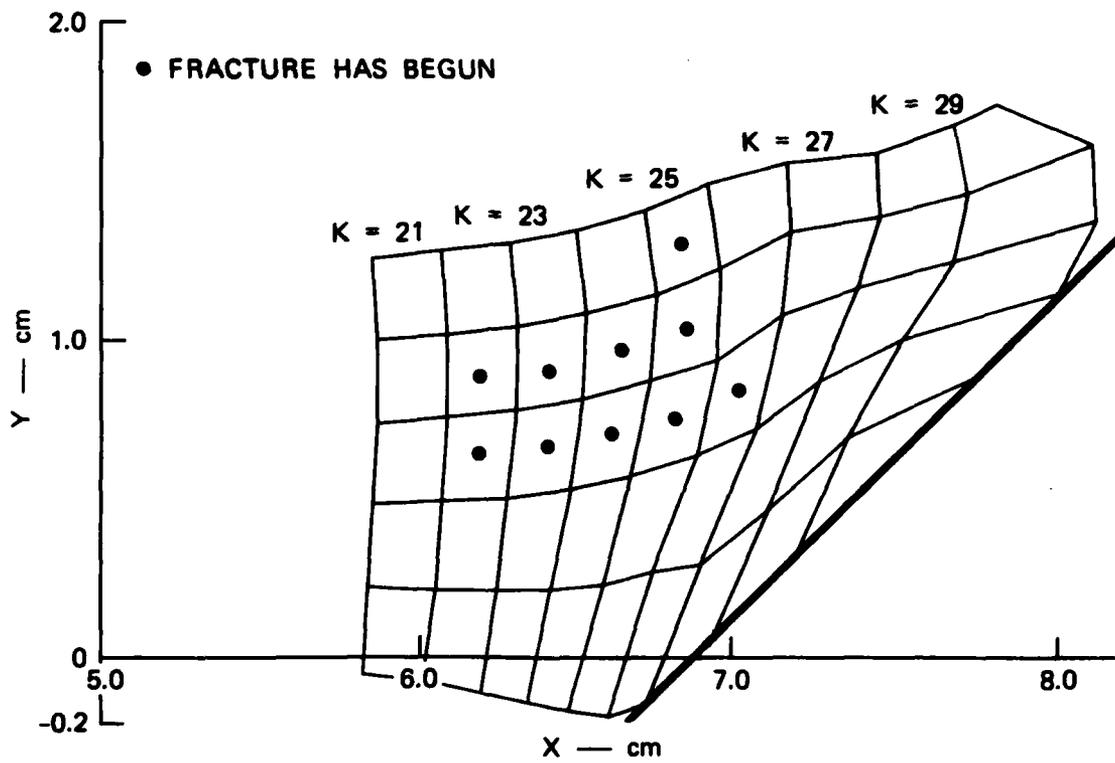
FIGURE 34   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 45°
OBLIQUITY, SLIPPING

81
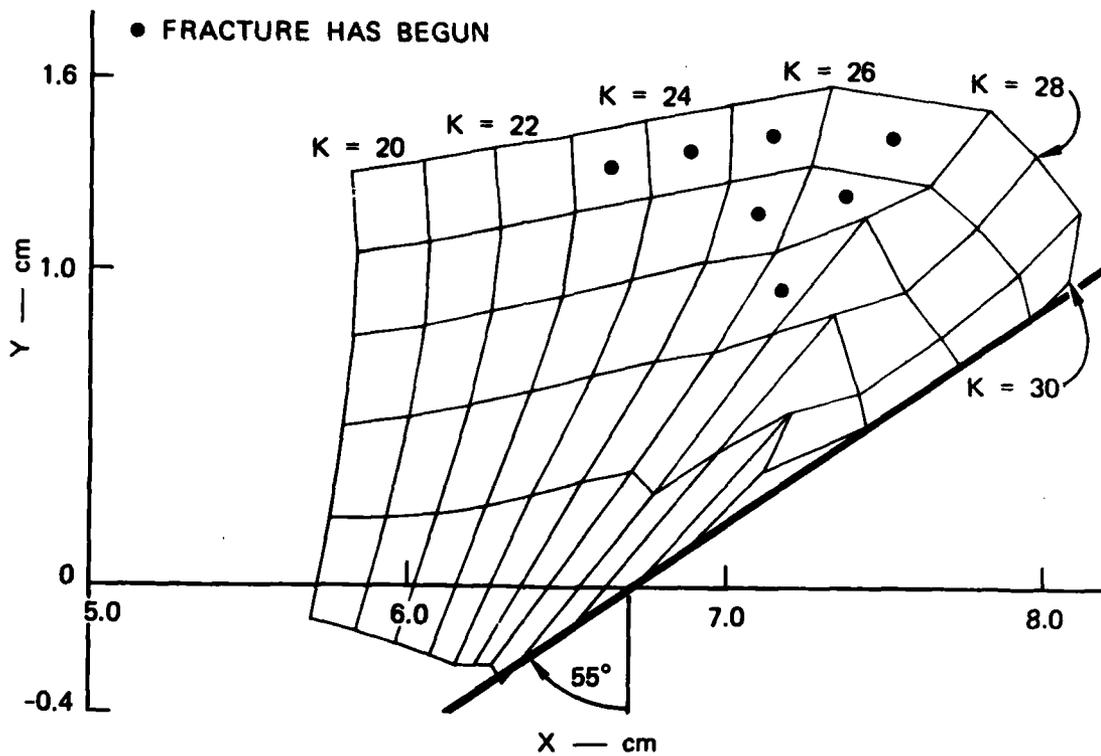
FIGURE 35   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 55°
OBLIQUITY, NO SLIPPING
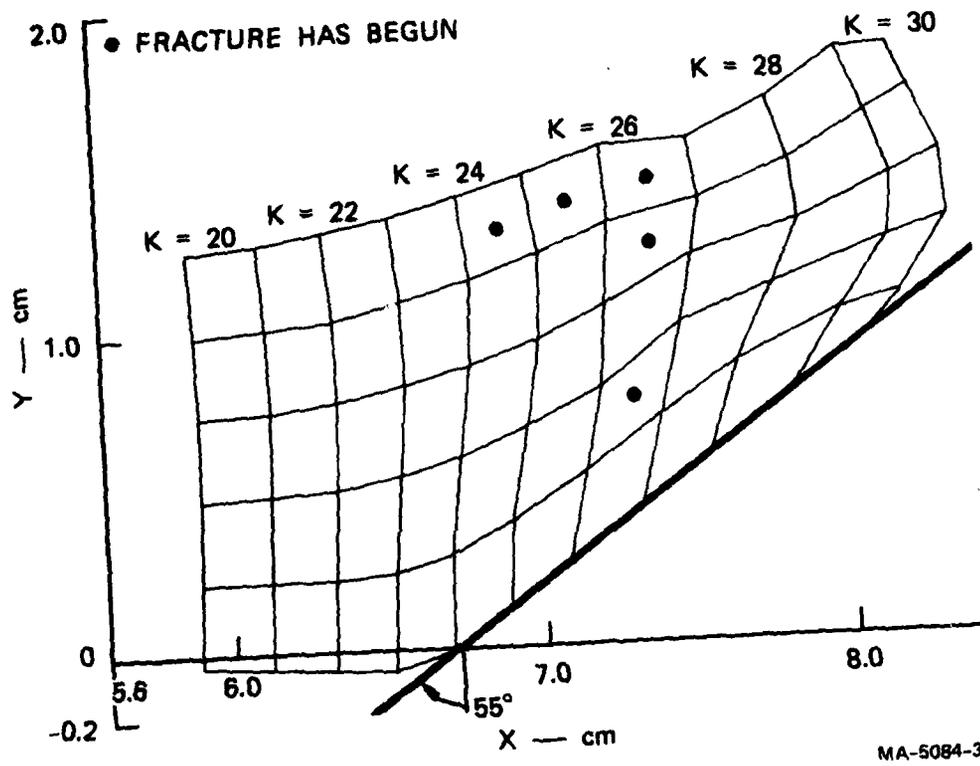
FIGURE 36   DISTORTION AND DAMAGE NEAR FRONT OF PROJECTILE PLATE FOR 55° OBLIQUITY, SLIPPING

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Graphs of the forces and bending moments on sections through the projectile at 14 μsec are shown in Figures 37 and 38. These values are computed by summing stresses from cells with the same initial Lagrangian position. The force and moment distributions build up gradually with time toward the configurations shown in Figures 37 and 38, but they are fairly constant by 14 μsec. A comparison of forces in these figures shows that the forces have a higher amplitude and extend deeper into the projectile for the sticking case. The moments show a similar trend for the 45° and 55° impacts, but a contrasting trend is shown at 30°. These moment variations should be studied further with calculations performed for longer durations. The results also shed light on the SRI oblique impact experiments reported in the previous section. Comparison of Figures 32, 34, and 36 show that the shear strain in the projectile nose is dramatically less for the 55° (slipping) impact than for impacts at lower angles. Thus, shear banding should become less prevalent at higher impact angles.

Future work should include the shear banding model as well as the tensile fracture model, and would help us gain an understanding of the interaction of these competitive failure modes.
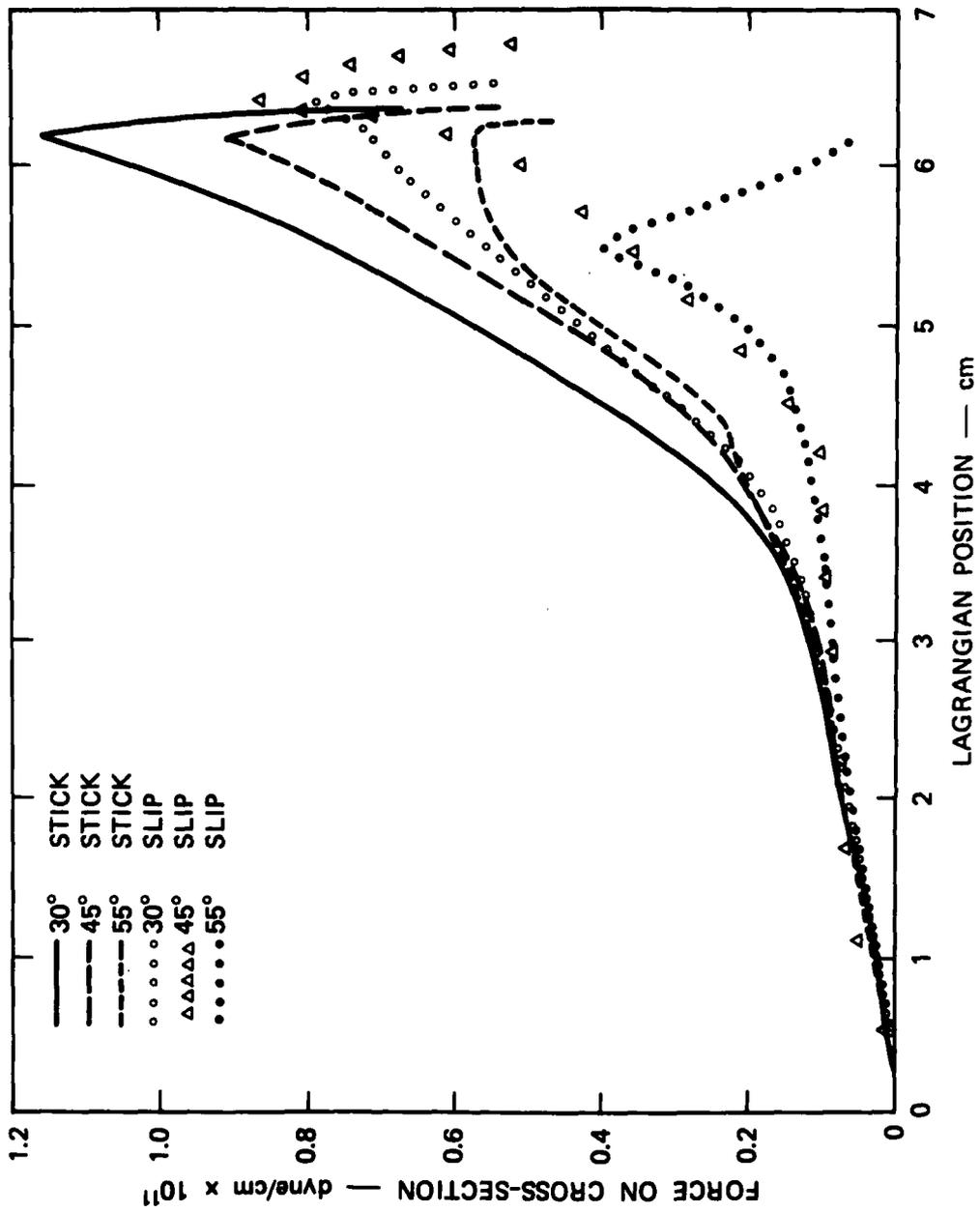
FIGURE 37   FORCE TRANSMITTED BY THE IMPACTING PLATE AS A FUNCTION OF DISTANCE ALONG THE PLATE

MA-5084-38

85
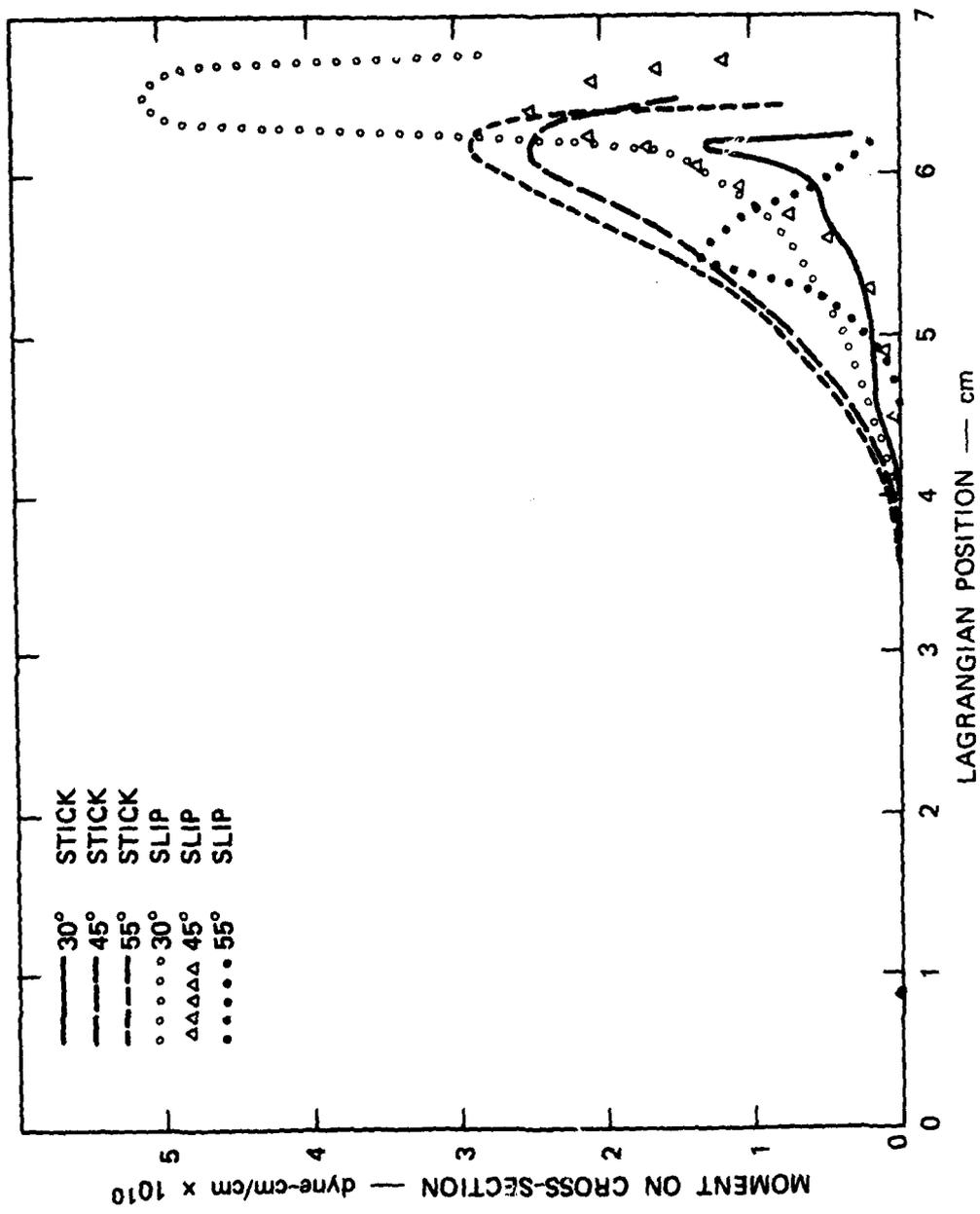
FIGURE 38   BENDING MOMENTS IN THE IMPACTING PLATE AS A FUNCTION OF DISTANCE ALONG
THE PLATE

MA-5084-39

APPENDIX A

EQUIVALENT PLASTIC STRAIN FOR A RADIALLY EXPANDING CYLINDRICAL TUBE

Consider a cylindrical tube, loaded by an internal pressure, expanding radially outward with no motion in the axial direction. What is the equivalent plastic strain, averaged across the wall thickness, that the cylinder has undergone after expansion and after the internal pressure has been relieved?

Consider the portion of a cylindrical tube shown in Figure 39. Let $r \equiv$ radius of the inner surface, $R \equiv$ radius of outer surface, $a = R-r \equiv$ wall thickness, and $x = \frac{1}{2}(R+r) \equiv$ radius of the midpoint. Furthermore let subscripts o and f refer to the original and final dimensions, respectively.

Our assumptions are that there is no axial strain ($\epsilon_z = 0$) and that the final volume strain is zero $\left(\epsilon_r + \epsilon_\theta + \epsilon_z = 0\right)$. Therefore, $\epsilon_r = -\epsilon_\theta$, and furthermore, the cross-sectional area of the tube remains constant $\left(\text{i.e., } a_o x_o = a_f x_f, \text{ or } \dfrac{a_f}{a_o} = \dfrac{x_o}{x_f}\right)$.

Now we define the equivalent plastic strain:[*]

$$\bar{\epsilon}^P \equiv \sqrt{\frac{3}{2}} \cdot \sqrt{\left(\epsilon_r^P\right)^2 + \left(\epsilon_\theta^P\right)^2 + \left(\epsilon_z^P\right)^2}$$

where the superscript P on the strain terms denotes plastic strains,

---

[*] This definition of equivalent plastic strain coincides with the definition of the differential $d\bar{\epsilon}^P$ in Eq. 4 if the ratios $d\epsilon_\theta^P/d\epsilon_r$ and $d\epsilon_z^P/d\epsilon_r^P$ are constant throughout the flow. For $\epsilon_z = 0$, the ratios are constant.
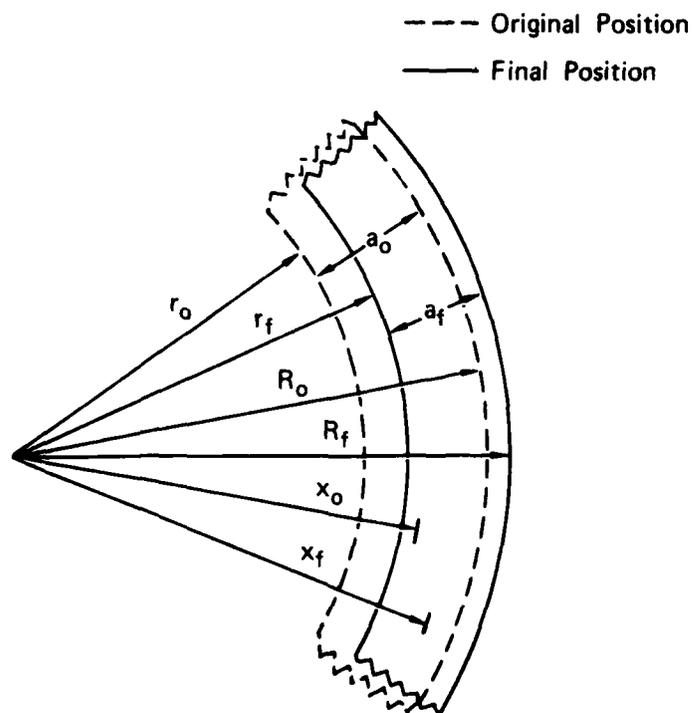
FIGURE 39   GEOMETRY AND NOMENCLATURE FOR EQUIVALENT
PLASTIC STRAIN DERIVATION

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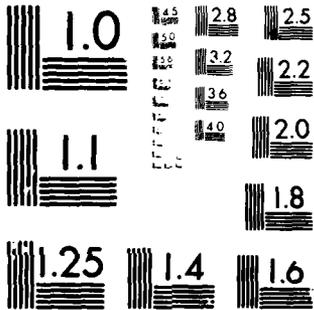
MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

which are equal to the final strains, since there are no residual
stresses. The strains may now be computed based on the change of
wall thickness, or on the change of radius. A change in thickness
causes a change in radial strain, $d\epsilon_r = da/a$. Then

$$\epsilon_r = \int_{a_o}^{a_f} \frac{da}{a} = \ln\left(\frac{a_f}{a_o}\right) = \ln\left(\frac{R_f - r_f}{R_o - r_o}\right)$$

Similarly a change in radius alters the circumference and induces a
circumferential strain $d\epsilon_\theta = dr/r$.

$$\epsilon_\theta = \frac{dr}{r} = \ln\left(\frac{x_f}{x_o}\right) = \ln\left(\frac{R_f + r_f}{R_o + r_o}\right)$$

In these expressions ϵ_r and ϵ_θ represent average strains through the wall
thickness. The equivalent plastic strain is then

$$\bar{\epsilon}^P = \sqrt{3}\,|\epsilon_r|$$

$$= \sqrt{3}\,|\epsilon_\theta|$$

For intact material the strains based on thickness or radius changes are
equivalent. But for fracturing material, the thickness measurement
gives the homogeneous plastic strain, whereas the radius gives the total
plastic strain (homogeneous plus that takes by the bands).

USE OF THE SHEAR BANDING SUBROUTINE IN WAVE
PROPAGATION CALCULATIONS

The shear band subroutine SHEAR2 is used in one- and two-dimensional
wave propagation codes to predict shear banding and fracture under high
rate loadings. The procedure for inserting the subroutine into wave
propagation codes is described below. A brief outline of the subroutine
is given, but the basic equations are derived in Chapter IV. The shear
banding parameters are described and some guidance is given for choice
of values. The subroutine input data and output are described and samples
are given, together with a listing of the subroutine.

1. Insertion Procedure

A wave propagation code normally has four main categories of operations:
reading the input data, initializing a finite difference grid, performing
calculations for each time increment at each grid point, and printing
the computed information. The subroutine SHEAR2 is written to correspond
with this organization; it has parts for reading data and initializing,
for calculating stresses and damage, and for printing the results. The
part of SHEAR2 to be used at each CALL is indicated by the parameter
NCALL. NCALL can have the following values:

0   Read data, initialize, and choose the weighting distribution
    of crack orientations in the solid angles according to the
    value of NANG, which is read in the BFR array.

1   Read data and initialize only for one material.

2   Calculate only.

3   Calculate and print.

4   Print only.

For each material, SHEAR2 must be called once to read data and
initialize variables. For the first of these calls, NCALL = 0; for
subsequent ones, NCALL $\overset{>}{=}$ 1. This CALL could appear anywhere in the
initialization phase of the calculations; it occurs in INPUT3 of TOODY
and in GENRAT of PUFF. These CALLS are placed so that the SHEAR2 data
appear with the other data for the material. SHEAR2 can then be called
for stress calculations alone, printing alone, or calculations and printing
together. These latter CALLs should appear after the strain increment
has been determined for each cell. In TOODY3, the second call is for
calculation and printing and is within the loops over each cell and each
time increment in the main program. In PUFF the computing and printing
CALL is in HSTRESS, the subroutine that controls all stress calculations.
There NCALL is normally set to 2, but on print cycles, NCALL is set to
3. The print CALL is made in conjunction with the computing CALL so
that certain temporary variables that are computed can also be printed
without the necessity of storing them.

The formal parameters of the subroutine are listed below in the
order that they appear in the CALL statement:
SUBROUTINE SHEAR2 (NCALL, IN, M, K, J, IH3, SX, SY, SZ, SXY, P, TAU,
DH, DOLD, DTO, EH, EOLD, EN, EMELT, EP, EX, EY, EZ, EXY, F, YHL, PLEN,
ROT, DROT, ESC, CN).

Only the first three parameters are required for the calls in which NCALL = 0 or 1. The meaning of the parameters is given in Tables 3 and 4. Some parameters are further described in Section 4 of this Appendix.

2. Outline of the Subroutine

The subroutine is written in three parts for reading, computing stresses and damage, and printing. In the reading portion, the BFR and NSIZE arrays are read, NSIZT and the two sign factors SS and SSE are set, and the weighting factors FNUC are initialized.

In the second part, the stresses and damage are computed. First, the T_z array is constructed from the existing damage quantities. The T_z values are used to transform the external stresses and strains to internal values. Then new stresses are computed on an elastic basis. If yielding occurs, the equivalent plastic strain, plastic work, and plastic shear strains in each band orientation are computed. Then existing bands are allowed to grow in proportion to the plastic shear strain. If some strain remains after growth, nucleation of new bands is computed. Then T_z is recomputed. Finally the internal stresses are transformed back to the external sign convention and orientation. A special zero-stress route is provided if fragmentation has occurred.

The final part of the subroutine contains the printing. Printing occurs only if there has been some nucleation of shear bands in the cell. This output is described in Section 6 of the Appendix.

3. Descriptions of Parameters

All the formal parameters and input quantities are defined in Tables 3 and 4. The subroutine calculations and variable names closely follow the derivation in Chapter IV so that internal variables should be

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Table 3

DEFINITION OF FORMAL PARAMETERS IN SHEAR2

NCALL	An indicator for the type of calculations required; see discussion in part 2 of this Appendix
IN	Data input file number (usually 5)
M	Material number
K	Cell number in the K direction
J	Cell number in the J direction
IH3	A damage indicator (20*TAU+2.9), output only; initialized at 2. Set to 25 at full separation.
SX	Deviator stress in the X direction (dyn/cm^2)
SY	Deviator stress in the Y direction (dyn/cm^2)
SZ	Deviator stress in the Z direction (dyn/cm^2)
SXY	Shear stress (dyn/cm^2)
P	Pressure calculated for the cell (dyn/cm^2)
TAU	Damage ($\Sigma\,NR^3$) in the cell calculated on the last cycle; ranges from 0 to 1
DH	New incoming density (g/cm^3)
DOLD	Cell density determined on the last cycle (g/cm^3)
DTO	Time step (sec)
EH	Internal energy of the cell in the present cycle (erg/g)
EOLD	Energy of cell in previous cycle (erg/g)
EN	Total number of shear bands in the cell (number/cm^3)
EMELT	Value of internal energy at which material will melt
EP	Total effective plastic strain
EX	Strain increment in the X direction
EY	Strain increment in the Y direction
EZ	Strain increment in the Z direction

EXY	Tensor shear strain increment
F	Thermal softening factor; varies from 0 to 1
YHL	Yield value calculated on last cycle (remains unchanged (1) if the yield has not been exceeded, or (2) if it has been exceeded and work hardening is zero) (dyn/cm^2)
PLEN	Work devoted to plastic strain on previous cycle (erg/g)
ROT	Total angle of cell rotation as of the previous cycle
DROT	Additional angle of cell rotation since the last cycle
ESC	Equation of state variables (array (6,20) must be dimensioned in the calling routine; some of the elements of this array are not used in SHEAR2)
ESC(M,1)	Original density (g/cc)
ESC(M,2)	Bulk modulus, $C(dyn/cm^2)$
ESC(M,3), ESC(M,4)	D and S in the pressure equation:

$$P = C\mu + D\mu^2 + S\mu^3, \text{ where } \mu = DH/ESC(M,1) - 1.$$

ESC(M,5)	Shear modulus (dyn/cm^2)
ESC(M,9)	Grüneisen's ratio,
CN	Array contains numbers of shear bands, band lengths, and other damage factors for each cell. The number of array elements required for each cell is $2\sum_1 NSIZE(i) + NANG$; array must be dimensioned in the calling routine.

Table 4

INPUT DATA FOR SHEAR2

BFR array	Nucleation and growth parameters for shear bands
BFR(M,22)	Growth coefficient, C_G
BFR(M,23)	Growth threshold, ϵ_{go} (unused)
BFR(M,24)	Nucleation size parameter, R_n in the relation $N_g = N_o \exp(-R/R_n)$ (cm)
BFR(M,25)	Nucleation rate coefficient, C_N (sec^2/cm^3)
BFR(M,26)	Nucleation threshold, ϵ_{no}
BFR(M,27)	Ratio of displacement to band radius, b
BFR(M,28)	Maximum nucleation band size (cm)
BFR(M,29)	Ratio of successive intervals between band sizes
BFR(M,30)	Time constant for deviator stress relation, T (sec)
BFR(M,31)	Work hardening modulus, Y_D (dyn/cm^2)
BFR(M,32)	Number of orientations used, NANG
BFR(M,33)	Thermal conductivity ($erg/sec/cm^2$) (unused)
BFR(M,34)	Specific heat at constant volume ($erg/g/^oK$) (unused)
BFR(M,35)	Critical internal energy required in the shear banding material to permit motion of the band, E_{cr} (erg/g)
NSIZE array	Number of size groups for each orientation

identifiable. However, some additional description is given here of
the parameters involved in the size groups and of the nucleation threshold.

BFR(M,32) = NANG is the number of orientations that may contain
shear bands. We have chosen six orientations for the present cylindrically
symmetric and axisymmetric problems. In some problems fewer could be
chosen if some orientations would not be activated because of symmetry.
The six chosen are for bands whose planes are normal to the radial,
circumferential, and axial directions, and at 45° between radial and
circumferential, between radial and axial, and between circumferential
and axial. Since only the latter three are active in the cylindrically
symmetric cases, NSIZE was set to zero for the first three orientations.
The direction of most interest is at 45° between radial and circumferential.
Probably satisfactory results would be obtained for the cylinders if
only that orientation were provided. Currently only six orientations
are fully implemented in the subroutine. More are probably not needed
until complete 3D problems are attempted.

In each orientation a number of size groups is provided. The
number of size groups, NSIZE, should be large (say 8 or 10) if considerable
detail is desired in the shear band size distribution for that orientation;
however, NSIZE could be 2 or 3 for less important orientations. For
all shear bands in a material there is a maximum band size for nucleation,
BFR(M,28). The choice of maximum band size should be larger than needed
so that the results do not significantly depend on this choice. In a
shear band calculation if NR^3 from the largest size group is a small
fraction of ΣNR^3, then the computed results are not dependent on the
largest radius size selected.

The radii of the bands are initialized so that the ratio of successive
intervals between radii is equal to BFR(M,29) = X, an input quantity.
Then if the maximum radius is R_N = BFR(M,28), the $(N-1)^{th}$ radius is

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$$R_{N-i} = R_N (1 - X^{N-i})/(1 - X^N).$$

The values of NSIZE and BFR(M,29) should be coordinated to give an accurate treatment of the band size distribution. A small ratio of say 1.2 to 1.6 could be used with large values of NSIZE. A small NSIZE would require a large BFR(M,29) to span the range of radii.

The threshold plastic shear strain is BFR(M,26), the value of strain that must occur in each orientation before nucleation can begin there. The threshold does not apply to the equivalent plastic strain, $\bar{\epsilon}^p$, but it applies to each element of the tensor $\epsilon^{ps}_{\varphi\theta}$. We note that $\epsilon^{ps}_{\varphi\theta}$ is only the shearing portion of the deviatoric plastic strain. This shearing strain is obtained from the flow rule or proportionality between plastic strain rate and deviator stress.

$$d\epsilon^p_{ij} = \sigma'_{ij} d\lambda$$

By squaring this equation and summing contributions from all the ij combinations, we obtain

$$\Sigma d\epsilon^p_{ij} d\epsilon^p_{ij} = (d\lambda)^2 \Sigma \sigma'_{ij} \sigma'_{ij}$$

But these sums are proportional to the squares of the effective stress and strain quantities ($\bar{\sigma}$ and $d\bar{\epsilon}^p$) defined in Equations 4 and 5. Hence

$$d\lambda = d\bar{\epsilon}^p/\bar{\sigma}$$

so the flow rule can be written

$$d\bar{\epsilon}^p_{ij} = d\bar{\epsilon}^p \frac{\sigma'_{ij}}{\bar{\sigma}} \quad \text{or} \quad d\bar{\epsilon}^{ps}_{ij} = d\bar{\epsilon}^p \frac{\sigma'^s_{ij}}{\bar{\sigma}}$$

where $\bar{\sigma}$ is the equivalent or von Mises shear stress and σ'^s_{ij} is the shearing component of the deviator stress. This shear component

is found by taking the vector difference between the total deviator S and the normal component S_n:

$$\sigma'^s_{ij} = \sqrt{S^2 - S_n^2} \ .$$

In general, S and S_n are defined as

$$S = \left(S_x^2 + S_y^2 + S_z^2\right)^{\frac{1}{2}}$$

$$S_n = \ell \, S_x + m \, S_y + n \, S_z$$

$$S_x = \ell \, \sigma_x + m \, \sigma_{yx} + n \sigma_{xz}$$

$$S_y = \ell \, \sigma_{xy} + m \, \sigma_y + n \, \sigma_{yz}$$

$$S_z = \ell \, \sigma_{xz} + m \sigma_{yz} + n \, \sigma_z$$

where σ_x, σ_y, σ_z, σ_{xy}, σ_{xz}, σ_{yz} are the stress tensor components and ℓ, m, n are direction cosines to the plane of interest. S_x, S_y, S_z are x, y, z components of the stress acting on the plane. A standard mechanics text should be referred to for details of this transformation.

As a guide in selecting values of BFR(M,26) based on measured thresholds for $\bar{\epsilon}^P$, consider the case where $d\epsilon_2 = -\, d\epsilon_1$ and $d\epsilon_3 = 0$. This strain configuration is approximated in the cylindrically symmetric calculations. For this case the shear strain at 45° between the directions 1 and 2 is $d\epsilon_{12}^{ps} = d\bar{\epsilon}^P / \sqrt{3}$. Hence, for an observed threshold of $\bar{\epsilon}^P = 0.28$, the appropriate value of BFR(M,26) is $0.28 / \sqrt{3} = 0.16$.

The major new material parameters are C_G, R_n, C_n, ϵ_{no} and b; each represents a well-defined physical process or quantity. From the present

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computed results, it appears that C_n is the most important for determining the shape of the fragment size distribution.

4. Transformations

As they enter the subroutine, all stress and strain quantities undergo two transformations--one in orientation and one in sign. The orientation transformation allows all computations in the subroutine to occur in a coordinate system that rotates with the material. (The shear band size groups and the cumulative plastic strain tensor also rotate with the material.) The sign transformation occurs so that the internal sign convention need not match the sign convention of the calling program.

Material rotation is provided by the calling program with two variables: ψ and $\Delta\psi$ (positive counterclockwise). The ψ is cumulative and must be stored external to the subroutine. Transformations use the angle $\psi' = \psi + \Delta\psi/2$. Stresses are transformed from σ_x, σ_y, σ_z, σ_{xy} to the starred quantities inside the subroutine by the following equations:

$$\sigma_x^* = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\psi' + \sigma_{xy}\sin 2\psi' \tag{36}$$

$$\sigma_y^* = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\psi' - \sigma_{xy}\sin 2\psi' \tag{37}$$

$$\sigma_z^* = \sigma_z \tag{38}$$

$$\sigma_{xy}^* = -\frac{\sigma_x - \sigma_y}{2}\sin 2\psi' + \sigma_{xy}\cos 2\psi' .$$

At the end of the calculation the internal stresses are rotated back to the external configuration. The equations for this final transformation are the same as Eqs. 36 to 38 with the starred and unstarred quantities interchanged and ψ' changed to $-\psi'$.

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The strain increments are also rotated to the material directions, and the same transformation equations are used for strain as for stress.

The sign transformation is made to account for a different sign convention. The internal convention is positive in compression for stress, pressure, and strain. If the convention matches the external one, SS and SSE (for stress and strain, respectively) are set to +1. If the external convention is that tension is positive, then the appropriate factor, SS or SSE is set to -1 in the DATA statement in SHEAR2. It is assumed that pressure is always positive in compression.

5. Output of the Subroutine

A sample of the output from the subroutine for four cells is given in Figure 40. This output is from a PUFF run with cylindrical symmetry in which orientations 4, 5, and 6 are activated. The parameters have the following meanings.

CN — Cumulative number of bands in an orientation. The bands at each radius are stored and used in computations, but the cumulative values are printed for ease in examining fragment size distributions.

CL — Shear band radius

TAUZ — ΣNR^3 in each orientation, summed over size groups

TAU — $\Sigma TAUZ$ summed over all orientations

EP — $\Sigma \Delta \bar{\epsilon}^p$ imposed on the material, summed over time.

Cumulative plastic strain — $\Sigma \Delta \epsilon^{ps}_{\varphi\theta}$ in each orientation, summed over time.

EN — Total number of shear bands in all orientations.

K, J — Coordinate designators, used for printout only.

```
K=  17 J=  17 IM3=  25 ROT= 0.        EN= .531E+01 TAU= .102E+01 EP= .998E+00
NG=   4
CN= .724E+00  .438E+00  .242E+00    .114E+00  .435E-01  .125E-01  .244E-02  .278E-03
CL= .371E+00  .815E+00  .121E+01    .153E+01  .176E+01  .197E+01  .219E+01  .243E+01
NG=   5
CN= .846E+00  .257E+00  .140E+00    .717E-01  .301E-01  .940E-02  .195E-02  .229E-03
CL= .307E+00  .420E+00  .646E+00    .836E+00  .989E+00  .114E+01  .132E+01  .157E+01
NG=   6
CN= .373E-01  .120E+01  .288E+00    .532E-01  .988E-02  .218E-02  .387E-03  .400E-04
CL= .102E+00  .237E+00  .367E+00    .496E+00  .645E+00  .787E+00  .874E+00  .966E+00
TOT PL STRAIN = 0.000000  0.000000   0.000  .280  .324
TAUZ= 0.000000  0.000000  .690987  .000057  .000018

K=  18 J=  18 IM3=  25 ROT= 0.        EN= .186E+03 TAU= .117E+01 EP= .105E+01
NG=   4
CN= .550E+00  .326E+00  .186E+00    .917E-01  .363E-01  .107E-01  .213E-02  .244E-03
CL= .400E+00  .822E+00  .122E+01    .164E+01  .194E+01  .216E+01  .238E+01  .260E+01
NG=   5
CN= .134E+02  .408E+00  .920E-01    .444E-01  .185E-01  .586E-02  .125E-02  .151E-03
CL= .215E+00  .266E+00  .313E+00    .380E+00  .473E+00  .603E+00  .785E+00  .104E+01
NG=   6
CN= .172E-03  .202E-02  .195E-01    .108E+00  .101E-01  .221E-02  .437E-03  .498E-04
CL= .956E-01  .153E+00  .190E+00    .228E+00  .268E+00  .319E+00  .390E+00  .490E+00
TOT PL STRAIN = 0.000000  0.000000   .282  .355  .268
TAUZ= 0.000000  0.000000  .690987  .000057  .000018

K=  19 J=  19 IM3=  25 ROT= 0.        EN= .211E+01 TAU= .102E+01 EP= .973E+00
NG=   4
CN= .391E+00  .217E+00  .123E+00    .624E-01  .257E-01  .790E-02  .162E-02  .190E-03
CL= .557E+00  .105E+01  .149E+01    .187E+01  .230E+01  .260E+01  .282E+01  .305E+01
NG=   5
CN= .512E+00  .108E+00  .491E-01    .219E-01  .804E-02  .225E-02  .433E-03  .493E-04
CL= .429E+00  .621E+00  .739E+00    .830E+00  .914E+00  .102E+01  .116E+01  .136E+01
NG=   6
CN= .121E+01  .395E+00  .978E-01    .210E-01  .540E-02  .148E-02  .292E-03  .329E-04
CL= .273E-01  .656E-01  .119E+00    .194E+00  .299E+00  .446E+00  .652E+00  .864E+00
TOT PL STRAIN = 0.000000  0.000000   .284  .337  .256
TAUZ= 0.000000  0.000000  .690987  .000057  .000018

K=  20 J=  20 IM3=  17 ROT= 0.        EN= .479E+00 TAU= .736E+00 EP= .814E+00
NG=   4
CN= .266E+00  .136E+00  .667E-01    .320E-01  .131E-01  .408E-02  .862E-03  .104E-03
CL= .576E+00  .123E+01  .172E+01    .211E+01  .247E+01  .285E+01  .317E+01  .343E+01
NG=   5
CN= .153E+00  .923E-01  .502E-01    .236E-01  .90RE-02  .242E-02  .507E-03  .563E-04
CL= .217E-01  .521E-01  .946E-01    .154E+00  .237E+00  .354E+00  .518E+00  .746E+00
NG=   6
CN= .587E-01  .384E-01  .222E-01    .109E-01  .435E-02  .131E-02  .264E-03  .301E-04
CL= .926E-02  .222E-01  .404E-01    .658E-01  .101E+00  .151E+00  .221E+00  .319E+00
TOT PL STRAIN = 0.000000  0.000000   .293  .248  .220
TAUZ= 0.000000  0.000000  .735995  .000349  .000013
```

FIGURE 40 SAMPLE OUTPUT FROM SHEAR2 FOR FOUR CELLS CONTAINING SHEAR BAND DAMAGE

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6. Input Data

Four sets of input data are provided in Figure 41 as samples of the use of the subroutine SHEAR2. All were used with SRI PUFF8 in the one-dimensional cylindrical-symmetry mode.

In Shot 5084-3, the explosive pressure is simulated by an exponentially decaying pressure on the inner boundary. The expression used is $P = P6* \exp(T/T6)$, where P is applied pressure and T is time. The parameters P6 (initial pressure) and T6 (time constant) are given in the NAMELIST statement at the end of the deck. For Shot 5084-4, the low density PETN explosive is modeled by a pressure-volume relation given in tabular form. The two experiments of Crowe[4] contain COMP B explosive, which is treated by a polytropic gas law following a simultaneous detonation. Input units are in the dyn, cm, g, sec CGS system.

Each data set contains three parts. First is the general running and printing information including JCYCS and TS, the number of computational cycles, and the problem time at which the calculation is halted. In the second part, the data for each material is given. The EQST card contains the Mie-Grüneisen quantities C, D, E_{sub}, Γ, H, and S where C, D, and S are the Hugoniot coefficients; E_{sub} is the sublimation energy; and Γ and H are Grüneisen's ratio in the solid and vapor states. The SH cards contain the 14 BFR numbers in order. Yield strength and shear modulus are on the YIELD card.

The layout of the materials and cells is handled in the final set of cards: NLAYER is the number of layers, and JMAT gives the material number in each layer. After the NLAYER card is a card for each layer describing the size and number of cells in that layer. In these samples the cell sizes are uniform in each layer.

7. Listing of SHEAR2

The subroutine SHEAR2 is listed in Figure 42.

```
IDENT FRAG RND FOR ERLICH-3
NTEDT =              0 NJEDIT =             2                    NALPHA =              2
JEDITS = ... 1   3_   5    7    9   11   12  .13   14   15   18   22   27   30
JEDIT 2    S2,P,COM3, 1.1,1.3,1.5,1.7
NEDIT =             10 JCYCS =           300 CKS =       3.000E+01 TS =        8.000E-05
NMTRLS =             4 MATFL =            -2 UZERO =    0.          NSCRB =  0000111111
HF-1 SH BAND         RHOS =     7.85E0     CFP=   030 DPY=   002 NVAR = 58 NCON=     0
EQST=     1.589E+12 5.170E+12 7.360E+10 1.69E0       0.25E0    5.170E+13
SH2       3.000E+01 .2000E+00 1.100E-02 1.000E-03 0.17       0.07UE+00 0.07CE+00
          1.4       3.000E-08 3.000E+08          6.          .?         .17 7.000E+09
NSIZE         0      0      0    8    8    8    0    0    0
YIELD =   1.030E+10 8.190E+11
MELT =    1.085E+10 1.000E+08 0.E0       9.975E-01 1.000E-01
PMMA-BKB (BARKER)    RHOS =     1.184E+00 CFP = 000 DPY = 001
EQST =    7.000E+10 4.050E+11 1.000E+10 1.000E+00 2.500E-01 39640E+11
YIELD =   1.000E+06 1.950E+10 2.850E+09
HF-1                 RHOS =     7.85E0     CFP=   000 DPY=   002              NCON=     0
EQST=     1.589E+12 5.170E+12 7.360E+10 1.69E0    _0.25E0    5.170E+13
YIELD =   1.030E+10 8.190E+11
MELT =    1.085E+10 1.000E+08 0.E0       9.975E-01 1.000E-01
LEAD (KOHN)          RHOS = __ 11.355     CFP = 000 DPY = 000
EQST =    5.00RE+11 4.986E+11 9.155E+09 2.2        0.25      2.019E+12
NLAYERS =             5 JMAT =        0    1    2    3    4
NZONES= 1      ...... 5 CELLS IN  1.512E+00 INCH
NZONES= 1             5 CELLS IN  8.750E-01 INCH
NZONES= 1             5 CELLS IN  0.500E+00 INCH
NZONES= 1            10 CELLS IN  2.500E+00 INCH
NZONES= 1             5 CELLS IN  1.000E+00 INCH
EXTRA
SNLIST P6(1)=81.5E9,T6(1)=-1.36E-5$
```

FIGURE 41   INPUT DATA FOR 4 CYLINDRICAL PUFF CALCULATIONS OF FRAGMENTING ROUNDS
            INCLUDING SHEAR BANDING

```
IDENT FRAG RND FOR ERLICH-4
NTEDT =            0  NJEDIT =        1                           NALPHA =            2
JEDIT =    S1,S2,COM3, 2,1,2,3,2,5,2,7
NEDIT =          100  JCYCS =       500  CKS =      3.000F+01  TS =        3.000E-04
NMTRLS =         5  MATFL =         1  UZERO =     0.          NSCRH =   000C111111
PETN             RHOS=       1.050E+00  CFP=   000 UPY=   060
EQST =     1.019E+11
IMAX =          21
RHO,P=    .100E+01    .501E+11    .794E+00    .300E+11    .633E+00    .183E+11
          .500E+00    .114E+11    .390E+00    .726E+10    .316E+00    .469E+10
          .251E+00    .309E+10    .200E+00    .207E+10    .159E+00    .140E+10
          .126E+00    .967E+09    .100E+00    .675E+09    .794E-01    .477E+09
          .633E-01    .341E+09    .600E-01    .845E+09    .398E-01    .182E+09
          .316E-01    .130E+09    .251E-01    .960E+08    .200E-01    .711E+08
          .159E-01    .530E+08    .126E-01    .397E+08    .100E-01    .299E+08

HF-1 SH BAND          RHOS =     7.85E0    CFP=   030 DPY=   002 NVAR = 50 NCON=        0
EQST=      1.589E+12 5.170E+12 7.360F+10 1.69E0     0.25E0     5.170E+13
SH2        3.000E+01 .2000E+00 1.100E-02 1.000E-03 0.17       0.070E+00 0.071F+00
           1.4       3.000E-08 3.000E+08         6.         .2         .17 7.000E+09
NSIZE          0     0     0     8     8     8     0     0     0
YIELD =    1.030E+10 8.190E+11
MELT =     1.085E+10 1.000E+06 0.E0        9.975E-01 1.000E-01
PMMA-BK8 (HANKER)  RHOS =     1.184E+00 CFP = 000 UPY = 001
EQST =     7.000E+10 4.050E+11 1.000E+10 1.000E+00 2.500E-01 39640E+11
YIELD =    1.000E+06 1.950E+10 2.850E+09
HF-1                  RHOS =     7.85E0    CFP=   000 DPY=   002              NCON=        0
EQST=      1.589E+12 5.170E+12 7.360E+10 1.69E0     0.25E0     5.170E+13
YIELD =    1.030E+10 8.190E+11
MELT =     1.085E+10 1.000E+06 0.E0        9.975E-01 1.000E-01
LEAD (KOHN)          RHOS =     11.355    CFP = 000 DPY = 000
EQST =     5.000E+11 4.466E+11 9.155E+09 2.2        0.25       2.019E+12
NLAYERS =          5  JMAT =         1     2     3     4     5
NZONES= 1            3 CELLS IN  1.512E+00 INCH
NZONES= 1            4 CELLS IN  8.750E-01 INCH
NZONES= 1            3 CELLS IN  1.125E+00 INCH
NZONES= 1            3 CELLS IN  2.500E+00 INCH
NZONES= 1            2 CELLS IN  1.000E+00 INCH
EXTRA
$NLIST U(2)=1.0,0.,1.0,0.,1.0,0.,1.0,0.,1.0$
```

FIGURE 41   INPUT DATA FOR 4 CYLINDRICAL PUFF CALCULATIONS OF FRAGMENTING ROUNDS
             INCLUDING SHEAR BANDING (Continued)

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```
IDENT FR 4 FRAGMTG RND OF HF-1 TO SIMULATE CROWES TESTS 1 AND 2.
NTEDT =              ñ NJEDIT =              1 NREZON =              0 NALPHA =              2
JEDIT =    S1,S2,COM3,.2,1,2.3,2.5,2,7,2.9
NEDIT =           10ñ JCYCS =            300 CKS =       30.E0      TS =      H.00.E-05
NMTRLS =             2 MATFL =              1 UZERO =    0.E0       NSCRH =
COMPB                 RHOS = ...1.72E0     CFP = 000  DPY = 012
EQST =    1.E0       0.E0       1.E0       1.841E0    1.841E0
QEXPL =   4.469E+1ñ
TENS =   -1.000E+09 0.E0    ....-1.E0.
MELT =   -1.E0
HF-1 SH BAND          RHOS =    7.85E0     CFP=  030  DPY=  002 NVAR = 58 NCON=    J
EQST=    1.589E+12 5.170E+12 7.360E+10 1.69E0    0.25E0    5.171E+13
SH2       3.000E+01 .2000E+00 1.100E-02 1.000E-03 0.17      0.070E+00 0.070E+00
          1.4       3.000E-08 3.000E+08           6.            .2      .17 7.000E+09
NSIZE       0    ñ    0    8    8    8    0    0    0
YIELD =   1.030E+10 8.190E+11
MELT =    1.085E+10 1.000E+UP 0.E0       9.975E-01 1.000E-01
NLAYERS =       2 JMAT      1    2
NZONES= 1          1ñ CELLS IN  1.5        INCH
NZONES= 1          1ñ CELLS IN  8.750E-01 INCH
```

```
IDENT FR 5 FRAG ROUND OF ARMCO IRON TO SIMULATE CROWES TESTS 3 AND 4
NTEDT =              ñ NJEDIT =              1 NREZON =              0 NALPHA =              2
JEDITS =   S1,S2,S3,COM3    2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9
NEDIT =           10ñ JCYCS =            300 CKS =       3.000E+01 TS =      H.00 E-05
NMTRLS =             2 MATFL =              1 UZERO =    0.
COMPB                 RHOS =    1.72.      CFP = 000  DPY = 012
EQST =    1.        0.         1.         1.841      1.841      0.          U.
QEXPL =   4.469E+1ñ
TENS =   -1.000E+09 0.      -1.
MELT =   -1.
ARMCO SH BAND          RHOS =    7.85E0     CFP=  030  DPY=  002 NVAR = 58 NCON=    0
EQST=    1.589E+12 5.170E+12 7.360E+10 1.69E0    0.25E0    5.170E+13
SH2       3.000E+01 .2000E+00 1.100E-02 3.000E-04 0.17      0.070E+00 0.070E+00
          1.4       3.000E-08 3.000E+08           6.            .2      .17 7.000E+09
NSIZE       0    ñ    0    8    8    8    0    0    0
MELT =    1.085E+1ñ 6.460E+09 5.700E-01 7.000E-01 1.850E-01
YIELD =   2.000E+09 8.190E+11
NLAYERS =       2 JMAT =      1    2
NZONES= 1          1ñ CELLS IN  1.5        INCH
NZONES= 1          1ñ CELLS IN  0.75       INCH
```

FIGURE 41   INPUT DATA FOR 4 CYLINDRICAL PUFF CALCULATIONS OF FRAGMENTING ROUNDS
            INCLUDING SHEAR BANDING (Concluded)

```
      SUBROUTINE SHEAR2(NCALL,IN,M,K,J,IM3,SX,SY,SZ,SXY,P,TAU,DM,UOLD,      SHEAR2    2
     1OTU,EM,EOLD,EN,EMELT,EP,EX,EY,EZ,EXY,F,YML,PLEN,ROT,DRUT,ESS,CN)       SHEAR2    3
                                                                            SHEAR2    4
C         ROUTINE FOR COMPUTATION OF STRESSES WITH RATE-DEPENDENT           SHEAR2    5
C      YIELD MODEL FOR DEVIATORS AND MIE-GRUNEISEN FOR PRESSURE.            SHEAR2    6
C      IF THRESHOLD PLASTIC STRAIN IS REACHED, SHEAR BANDS ARE             SHEAR2    7
C      NUCLEATED AND GROWN IN 6 ORIENTATIONS.                             SHEAR2    8
C         SX, SY, SZ ARE DEVIATORS IN EXTERNAL SIGN CONVENTION.           SHEAR2    9
C      P IS POSITIVE IN COMPRESSION.  INTERNAL SIGN CONVENTION IS         SHEAR2   10
C      POSITIVE IN COMPRESSION FOR ALL STRESS AND STRAIN QUANTITIES       SHEAR2   11
C      ST IS TOTAL STRESS AT PREVIOUS TIME.  SE IS NEW DEVIATOR.          SHEAR2   12
C      EX, EY, EZ, EXY ARE STRAIN INCREMENTS IN EXTERNAL SIGN CON.        SHEAR2   13
C      SS, SSE CHANGE EXTERNAL SIGN CONVENTION TO INTERNAL FOR            SHEAR2   14
C      STRESS AND STRAIN, RESPECTIVELY.                                    SHEAR2   15
C                                                                          SHEAR2   16
      DIMENSION BFR(6,35),NSIZE(30,9),FNUC(9),TAUZ(6),EFR(3),VFR(6),      SHEAR2   17
     1ST(4),ES(4),SE(4),TEP(6),ESC(6,20),NSIZT(6)                         SHEAR2   18
     2  ,CN(100),DEP(4),CLA(100),CNA(100),VMAX(6)                         SHEAR2   19
      EQUIVALENCE (CNA,CLA)                                                SHEAR2   20
C                                                                          SHEAR2   21
      DATA SS,SSE/-1.,-1./                                                 SHEAR2   22
      NC1=NCALL+1                                                          SHEAR2   23
      GO TO (10,10,100,100,900)NC1                                        SHEAR2   24
10    READ(IN,1002) A1,A2,(BFR(M,I),I=22,35)                              SHEAR2   25
      PRINT 1002, A1,A2,(BFR(M,I),I=22,35)                                SHEAR2   26
1002  FORMAT(2A5,7E10.3/10X,7E10.3)                                       SHEAR2   27
      READ(IN,1003)A1,A2,(NSIZE(M,I),I=1,9)                               SHEAR2   28
      PRINT 1003,A1,A2,(NSIZE(M,I),I=1,9)                                 SHEAR2   29
1003  FORMAT(2A5,14I5)                                                    SHEAR2   30
      VMAX(M)=0.                                                          SHEAR2   31
      NSIZT(M)=NSIZE(M,1)                                                 SHEAR2   32
      DO 14 I=2,9                                                         SHEAR2   33
14    NSIZT(M)=NSIZT(M)+NSIZE(M,I)                                        SHEAR2   34
      VFR(M)=1.                                                           SHEAR2   35
      IF (NCALL .EQ. 1)  GO TO 65                                         SHEAR2   36
      NANG=BFR(M,32)                                                      SHEAR2   37
      DO 16 I=1,3                                                         SHEAR2   38
      FNUC(I)=.111111                                                     SHEAR2   39
16    FNUC(I+3)=.222222                                                   SHEAR2   40
      IF(NANG - 6) 20,40,30                                               SHEAR2   41
20    FNUCI=.333333                                                       SHEAR2   42
      IF (NANG .GE.4) FNUCI =.25                                          SHEAR2   43
      DO 25 I=1,4                                                         SHEAR2   44
25    FNUC(I)=FNUCI                                                       SHEAR2   45
      IF (NANG .EQ.2) FNUC(2) =.6666667                                   SHEAR2   46
      IF (NANG .NE. 5)  GO TO 30                                          SHEAR2   47
      FNUC(4)=.125                                                        SHEAR2   48
      FNUC(7)=.125                                                        SHEAR2   49
      FNUC(5)=0.                                                          SHEAR2   50
      FNUC(6)=0.                                                          SHEAR2   51
      GO TO 40                                                            SHEAR2   52
30    DO 35 I=7,NANG                                                      SHEAR2   53
      FNUC(I)=.111111                                                     SHEAR2   54
35    FNUC(I-3)=.111111                                                   SHEAR2   55
40    CONTINUE                                                            SHEAR2   56
65    RETURN                                                              SHEAR2   57
C*********                                                                 SHEAR2   58
C                                                                          SHEAR2   59
C        COMPUTE STRESS AND DAMAGE                                         SHEAR2   60
C                                                                          SHEAR2   61
C*********                                                                 SHEAR2   62
```

FIGURE 42 LISTING OF SUBROUTINE SHEAR2

```
  100  IF (IH3 .GE. 25) GO TO 800                                    SHEAR2   63
       IF (VMAX(M) .EQ. 0.) VMAX(M)=SQRT(ESC(M,5)/ESC(M,1))          SHEAR2   64
   C     COMPUTE STRESS REDUCTION FACTORS TAUZ(I)                    SHEAR2   65
       TAU=0.                                                        SHEAR2   66
       JN=0                                                          SHEAR2   67
       DO 110 NG=1,NANG                                              SHEAR2   68
       TAUZ(NG)=0.                                                   SHEAR2   69
       IF(NSIZE(M,NG) .EQ. 0 .OR. CN(JN+1) .EQ. 0.)GO TO 110         SHEAR2   70
       NSIZEM=NSIZE(M,NG)                                            SHEAR2   71
       DO 120 I=1,NSIZEM                                             SHEAR2   72
       JNN=JN+2*I-1                                                  SHEAR2   73
  120  TAUZ(NG)=TAUZ(NG)+CN(JNN)*CN(JNN+1)**3                        SHEAR2   74
       TAU=TAU+TAUZ(NG)                                              SHEAR2   75
  110  JN=JN+2*NSIZE(M,NG)                                           SHEAR2   76
  C**********                                                        SHEAR2   77
  C          INITIAL TRANSFORMATION                                  SHEAR2   78
  C**********                                                        SHEAR2   79
  C          ADJUST SIGNS,ROTATE STRESS,TRANSFORM TO STRESS IN SOLID(ST) SHEAR2 80
       RT=ROT+ROT+OROT                                               SHEAR2   81
       EMU=DOLU/ESC(M,1)-1.                                          SHEAR2   82
       PH=EMU*(ESC(M,2)+EMU*(ESC(M,3)+EMU*ESC(M,4)))                 SHEAR2   83
       PS=PH*(1.-ESC(M,9)*EMU/2.)+DOLD*ESC(M,9)*EOLD                 SHEAR2   84
       IF(PS .GT. 0.) P=P-PS*TAU                                     SHEAR2   85
       SA=(SX+SY)*SS/2.+P                                            SHEAR2   86
       SOR=SIN(RT)                                                   SHEAR2   87
       COR=COS(RT)                                                   SHEAR2   88
       SB=((SX-SY)/2.*COR+SXY*SOR)*SS                                SHEAR2   89
       DO 140 I=1,4                                                  SHEAR2   90
  140  ST(I)=0.                                                      SHEAR2   91
       G2=2.*ESC(M,5)                                                SHEAR2   92
       ST(1)= (SA+SB)/AMAX1(0.02,(1.-(3.*TAUZ(1)+1.5*(TAUZ(4)+TAUZ(5))) SHEAR2 93
      1 *VFR(M)))                                                    SHEAR2   94
       ST(2)=(SA-SB)/AMAX1(0.02,(1.-(3.*TAUZ(2)+1.5*(TAUZ(4)+TAUZ(6))) SHEAR2 95
      1 *VFR(M)))                                                    SHEAR2   96
       ST(3)=(P-(SX+SY)*SS)/AMAX1(0.02,(1.-(3.*TAUZ(3)+1.5*(TAUZ(5)+ SHEAR2   97
      1 TAUZ(6)))*VFR(M)))                                           SHEAR2   98
       ST(4)=((SY-SX)/2.*SOR+SXY*COR)*SS/(1.-(1.5*(TAUZ(1)+TAUZ(2))+3. SHEAR2 99
      1 *TAUZ(4))*VFR(M))                                            SHEAR2  100
       P=(ST(1)+ST(2)+ST(3))/3.                                      SHEAR2  104
   C          ROTATE STRAINS TO BAND ORIENTATIONS                    SHEAR2  105
       EA=(EX+EY)/2.                                                 SHEAR2  106
       EB=(EX-EY)/2.*COR+EXY*SOR                                     SHEAR2  107
       EBAR=0.6667*(DH-DOLD)/(DH+DOLD)                               SHEAR2  108
       NSTEP=SQRT((ABS(EA)+ABS(EB)+ABS(EBAR))/.002)                  SHEAR2  109
       NSTEP=MAX0(NSTEP,1)                                           SHEAR2  110
       ES(1)= (EA+EB)*SSE/NSTEP                                      SHEAR2  111
       ES(2)= (EA-EB)*SSE/NSTEP                                      SHEAR2  112
       ES(3)=3.*EBAR/NSTEP-ES(1)-ES(2)                              SHEAR2  113
       ES(4)=(((EY-EX)/2.*SOR+EXY*COR)*SSE)/NSTEP                    SHEAR2  114
       DO 600 NS=1,NSTEP                                             SHEAR2  115
       DO 160 I=1,3                                                  SHEAR2  116
  160  SE(I)=ST(I)+G2*(ES(I)-EBAR/NSTEP)-P                           SHEAR2  117
       SE(4)= ST(4)+G2*ES(4)                                         SHEAR2  118
       SN=SQRT(1.5*(SE(1)**2+SE(2)**2+SE(3)**2+2.*SE(4)**2))         SHEAR2  119
       DHN=DOLU+FLOAT(NS)/FLOAT(NSTEP)*(DH-DOLD)                     SHEAR2  120
       EMU=DHN/ESC(M,1)-1.                                           SHEAR2  121
       PH=EMU*(ESC(M,2)+EMU*(ESC(M, 3)+EMU*ESC(M, 4)))               SHEAR2  122
       PE=PH*(1.-ESC(M, 9)*EMU          /2.)+DHN*ESC(M, 9)*EH        SHEAR2  123
       Y1=YML                                                        SHEAR2  124
       IF (SN .LT. Y1)  GO TO 500                                    SHEAR2  125
```

FIGURE 42   LISTING OF SUBROUTINE SHEAR2   (Continued)

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```
C**********
C          YIELD AND PLASTIC STRAIN CALCULATIONS                    SHEAR2    126
C**********                                                          SHEAR2    127
                                                                    SHEAR2    128
      EXPT = EXP(-DTO/BFR(M,30)/NSTEP)                              SHEAR2    129
      YEB=(Y1+BFR(M,31)*SN/2./G2)/(1.+BFR(M,31)/2./G2)/SN          SHEAR2    130
      DO 180  I=1,3                                                SHEAR2    131
 180  SE(I)=(ST(I)-P)*EXPT+(YEB*SE(I)+BFR(M,30)*(SE(I)-ST(I)+P)/   SHEAR2    132
     1DTO*NSTEP)*(1.-EXPT)                                         SHEAR2    133
      SE(4)=ST(4)*EXPT+(YEB*SE(4)+BFR(M,30)*(SE(4)-ST(4))/DTO*NSTEP)* SHEAR2  134
     1 (1.-EXPT)                                                   SHEAR2    135
      DO 200  I=1,3                                                SHEAR2    136
 200  DEP(I)=ES(I)-EBAR/NSTEP-(SE(I)-ST(I)+P)/G2                   SHEAR2    137
      DEP(4)=ES(4)-(SE(4)-ST(4))/G2                                SHEAR2    138
      DGAMMA=SQRT(1.5*(DEP(1)**2+DEP(2)**2+DEP(3)**2)+0.75*DEP(4)  SHEAR2    139
     1**2)                                                         SHEAR2    140
      YHL=YHL+BFR(M,31)*DGAMMA                                     SHEAR2    141
      DPLENR=((SE(1)+ST(1)-P)*DEP(1)+(SE(2)+ST(2)-P)*DEP(2)+(SE(3)+S SHEAR2   142
     1T(3)-P)*DEP(3)+(SE(4)+ST(4))*DEP(4))/2./DMN*AMAX1(0.,1.-TAU) SHEAR2    143
      DPLENR=ABS(DPLENR)                                           SHEAR2    144
      EP=EP+DGAMMA                                                 SHEAR2    145
      PLEN=PLEN+DPLENR                                             SHEAR2    146
C**********                                                          SHEAR2    147
C          COMPUTE PLASTIC STRAIN IN EACH ORIENTATION              SHEAR2    148
C**********                                                          SHEAR2    149
      STR1=ABS(SE(4))                                              SHEAR2    150
      STR2=ABS(SE(4))                                              SHEAR2    151
      STR3=0.                                                      SHEAR2    152
      STR4=ABS(SE(1)-SE(2))/2.                                     SHEAR2    153
      STR5=SQRT((SE(1)-SE(3))**2+2.*SE(4)**2)/2.                   SHEAR2    154
      STR6=SQRT((SE(2)-SE(3))**2+2.*SE(4)**2)/2.                   SHEAR2    155
      SN=SQRT(1.5*(SE(1)**2+SE(2)**2+SE(3)**2+2.*SE(4)**2))        SHEAR2    156
      TEP(1)=DGAMMA/SN*STR1                                        SHEAR2    157
      TEP(2)=DGAMMA/SN*STR2                                        SHEAR2    158
      TEP(3)=DGAMMA/SN*STR3                                        SHEAR2    159
      TEP(4)=DGAMMA/SN*STR4                                        SHEAR2    160
      TEP(5)=DGAMMA/SN*STR5                                        SHEAR2    161
      TEP(6)=DGAMMA/SN*STR6                                        SHEAR2    162
C**********                                                          SHEAR2    163
C          GROWTH PROCESS                                          SHEAR2    164
C**********                                                          SHEAR2    165
      NTOT=2*NSIZT(M)                                              SHEAR2    166
      DO 250  I=1,NTOT                                             SHEAR2    167
 250  CNA(I)=CN(I)                                                 SHEAR2    168
      IF (EN .EQ. 0) GO TO 360                                     SHEAR2    169
      JN=0                                                         SHEAR2    170
      DC=VMAX(M)*DTO/NSTEP                                         SHEAR2    171
      DO 350  NG=1,NANG                                            SHEAR2    172
      DGAM=0.                                                      SHEAR2    173
      IF (NSIZE(M,NG) .EQ. 0. .OR. CN(JN+1) .EQ. 0 .OR. TEP(NG) .LE. 0.) SHEAR2 174
     1 GO TO 345                                                   SHEAR2    175
      EXPE=EXP(BFR(M,22)*TEP(NG))                                  SHEAR2    176
      NSIZEM=NSIZE(M,NG)                                           SHEAR2    177
      DO 300 I=1,NSIZEM                                            SHEAR2    178
      JN2=JN+2*(NSIZE(M,NG)+1-I)                                   SHEAR2    179
      CLA(JN2)=AMIN1(CN(JN2)*EXPE,CN(JN2)+DC)                      SHEAR2    180
 300  DGAM=DGAM+CN(JN2-1)*3.14*BFR(M,27)*(CLA(JN2)**3-CN(JN2)**3)  SHEAR2    181
      IF(DGAM .LE. TEP(NG))GO TO 345                               SHEAR2    182
      RR=TEP(NG)/DGAM                                              SHEAR2    183
      DCR=DC*RR                                                    SHEAR2    184
      EXPE=EXPE**RR                                                SHEAR2    185
      NSIZEM=NSIZE(M,NG)                                           SHEAR2    186
      DO 340 I=1,NSIZEM                                            SHEAR2    187
      JN2=JN+2*(NSIZE(M,NG)+1-I)                                   SHEAR2    188
```

FIGURE 42   LISTING OF SUBROUTINE SHEAR2 (Continued)

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```
340   CLA(JN2)=AMIN1(CN(JN2)*EXPE,CN(JN2)+DCR)          SHEAR2   189
345   TEP(NG)=AMAX1(0.,TEP(NG)-DGAM)                    SHEAR2   190
350   JN=JN+NSIZE(M,NG)*2                              SHEAR2   191
360   CONTINUE                                         SHEAR2   192
      DO 365 NG=1,NANG                                 SHEAR2   193
365   CN(NTOT+NG)=CN(NTOT+NG)+TEP(NG)                  SHEAR2   194
C**********                                            SHEAR2   195
C           NUCLEATION PROCESS                         SHEAR2   196
C**********                                            SHEAR2   197
      TEPM=0                                           SHEAR2   198
      DO 370 NG=1,NANG                                 SHEAR2   199
370   TEPM=AMAX1(TEPM,CN(NTOT+NG)+TEP(NG))             SHEAR2   200
      IF (TEPM .LT. BFR(M,26))  GO TO 500              SHEAR2   201
      JN=0                                             SHEAR2   202
      DO 450  NG=1,NANG                                SHEAR2   203
      IF (NSIZE(M,NG) .EQ. 0)  GO TO 450               SHEAR2   204
      IF (CN(NTOT+NG)+TEP(NG) .LT. BFR(M,26) .OR. TEP(NG) .LT. 1.E-5)   SHEAR2   205
     1 GO TO 450                                       SHEAR2   206
      DNO=TEP(NG)*BFR(M,25)*FNUC(NG)*(DPLENR/DIO*NSTEP/BFR(M,35))**2    SHEAR2   207
      CNR=0.                                           SHEAR2   208
      NSIZEM=NSIZE(M,NG)                               SHEAR2   209
      DO 440 I=1,NSIZEM                                SHEAR2   210
      II=NSIZE(M,NG)+1-I                               SHEAR2   211
      JNI=JN+2*II                                      SHEAR2   212
      IF(CLA(JNI) .NE. 0.) GO TO 420                   SHEAR2   213
      CLA(JNI)=BFR(M,28)*(1.-BFR(M,29)**II)/(1.-BFR(M,29)**NSIZE(M,NG)) SHEAR2   214
      CN(JNI)=CLA(JNI)                                 SHEAR2   215
420   CNL=DNO*EXP(-(CLA(JNI)+CN(JNI))/2./BFR(M,24))    SHEAR2   216
      JNN=JN+2*II-1                                    SHEAR2   217
      CNA(JNN)=CNL-CNR+CN(JNN)                         SHEAR2   218
440   CNR=CNL                                          SHEAR2   219
      EN=EN+CNL                                        SHEAR2   220
450   JN=NSIZE(M,NG)*2+JN                              SHEAR2   221
470   CONTINUE                                         SHEAR2   222
C**********                                            SHEAR2   223
C           COMPUTE TAU AND REFILL MAIN ARRAYS         SHEAR2   224
C**********                                            SHEAR2   225
      TAU=0.                                           SHEAR2   226
      JN=0                                             SHEAR2   227
      IF(EN .EQ. 0.)  GO TO 500                        SHEAR2   228
      DO 490   NG=1,NANG                               SHEAR2   229
      TAUZ(NG)=0.                                      SHEAR2   230
      IF (NSIZE(M,NG) .EQ.0 .OR. CNA(JN+1) .EQ. 0.)  GO TO 490  SHEAR2   231
      NSIZEM=NSIZE(M,NG)                               SHEAR2   232
      DO 480 I=1,NSIZEM                                SHEAR2   233
      JNN=JN+2*I-1                                     SHEAR2   234
      CN(JNN+1)=CLA(JNN+1)                             SHEAR2   235
      CN(JNN)=CNA(JNN)                                 SHEAR2   236
480   TAUZ(NG)=TAUZ(NG)+CNA(JNN)*CLA(JNN+1)**3         SHEAR2   237
      TAU=TAU+TAUZ(NG)                                 SHEAR2   238
490   JN=JN+NSIZE(M,NG)*2                              SHEAR2   239
      IF(TAU*VFR(M) .GE. 1.) GO TO 800                 SHEAR2   240
500   CONTINUE                                         SHEAR2   241
      P=PE                                             SHEAR2   242
      DO 550 I=1,3                                     SHEAR2   243
550   ST(I)=SE(I)+P                                    SHEAR2   244
      ST(4)=SE(4)                                      SHEAR2   245
600   CONTINUE                                         SHEAR2   246
C**********                                            SHEAR2   247
C     TRANSFORMATION TO GLOBAL ORIENTATION             SHEAR2   248
C**********                                            SHEAR2   249
```

FIGURE 42   LISTING OF SUBROUTINE SHEAR2   (Continued)

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```
      ST(4)=ST(4)*AMAX1(0.,(1.-(1.5*(TAUZ(1)+TAUZ(2))+3.*TAUZ(4))*VFR(M)   SHEAR2   252
     1 ))                                                                   SHEAR2   253
      ST(1)=ST(1)*AMAX1(0.,(1.-(3.*TAUZ(1)+1.5*(TAUZ(4)+TAUZ(5)))*VFR(M)    SHEAR2   254
     1 ))                                                                   SHEAR2   255
      ST(2)=ST(2)*AMAX1(0.,(1.-(3.*TAUZ(2)+1.5*(TAUZ(4)+TAUZ(6)))*VFR(M)    SHEAR2   256
     1 ))                                                                   SHEAR2   257
      ST(3)=ST(3)*AMAX1(0.,(1.-(3.*TAUZ(3)+1.5*(TAUZ(5)+TAUZ(6)))*VFR(M)    SHEAR2   258
     1 ))                                                                   SHEAR2   259
      P=(ST(1)+ST(2)+ST(3))/3.                                             SHEAR2   260
      SA=(ST(1)+ST(2))/2.                                                  SHEAR2   261
      SB=(ST(1)-ST(2))/2.*COR-ST(4)*SOR                                    SHEAR2   262
      SXY=((ST(1)-ST(2))/2.*SOR+ST(4)*COR)*SS                             SHEAR2   263
      SX=(SA+SB-P)*SS                                                      SHEAR2   264
      SY=(SA-SB-P)*SS                                                      SHEAR2   265
      SZ=(ST(3)-P)*SS                                                      SHEAR2   266
      IF (PE .GT. 0.) P=P+PE*TAU                                          SHEAR2   267
      IH3=20.*TAU+2.9                                                     SHEAR2   268
      IF (NCALL .GE. 3) GO TO 900                                         SHEAR2   270
      RETURN                                                             SHEAR2   271
C***********                                                            SHEAR2   272
C     COMPLETE SEPARATION                                               SHEAR2   273
C***********                                                            SHEAR2   274
  800 EMU=DH/ESC(M,1)-1.                                                 SHEAR2   275
      PH=EMU*(ESC(M,2)+EMU*(ESC(M, 3)+EMU*ESC(M, 4)))                   SHEAR2   276
      PE=PH*(1.-ESC(M, 9)*(DH/ESC(M,1)-1.)/2.)+DH*ESC(M, 9)*EH          SHEAR2   277
      P=AMAX1(PE,0.)                                                     SHEAR2   278
      SX=0.                                                              SHEAR2   279
      SY=0.                                                              SHEAR2   280
      SZ=0.                                                              SHEAR2   281
      SXY=0.                                                             SHEAR2   282
      IH3=25                                                             SHEAR2   283
      IF (NCALL .GE. 3) GO TO 900                                        SHEAR2   284
      RETURN                                                             SHEAR2   285
C * * * * *                                                             SHEAR2   286
C                                                                       SHEAR2   287
C     PRINT DAMAGE ARRAYS                                               SHEAR2   288
C                                                                       SHEAR2   289
C * * * * *                                                             SHEAR2   290
  900 IF (EN .EQ. 0.) RETURN                                            SHEAR2   291
      PRINT 8000,K,J,IH3,ROT,EN,TAU,EP                                  SHEAR2   292
      JN=0                                                              SHEAR2   293
      DO 1000 NG=1,NANG                                                 SHEAR2   294
      NS=NSIZE(M,NG)*2                                                  SHEAR2   295
      IF (NS .EQ. 0 .OR. CN(JN+1) .EQ. 0.) GO TO 1000                   SHEAR2   296
      II=JN+NS-1                                                        SHEAR2   297
      CNA(II)=CN(II)                                                    SHEAR2   298
      IF (NS .LE. 2) GO TO 975                                          SHEAR2   299
      DO 970 I=3,NS,2                                                   SHEAR2   300
      II=NS-I+JN                                                        SHEAR2   301
  970 CNA(II)=CN(II)+CNA(II+2)                                          SHEAR2   302
  975 CONTINUE                                                          SHEAR2   303
      PRINT 8500,NG                                                     SHEAR2   304
      PRINT 9001,(CNA(JN+I),I=1,NS,2)                                   SHEAR2   305
      PRINT 9002,(CN(JN+I),I=2,NS,2)                                    SHEAR2   306
 8000 FORMAT(3H K=I5,3H J=I5,5H IH3=I5,5H ROT=E10.3,4H EN=E10.3,        SHEAR2   307
     1 5H TAU=E10.3,4H EP=E10.3)                                        SHEAR2   308
 8500 FORMAT(4H NG=I5)                                                  SHEAR2   309
 9001 FORMAT(4H CNm=10(E10.3,2X))                                       SHEAR2   310
 9002 FORMAT(4H CL=10(E10.3,2X))                                        SHEAR2   311
 1000 JN=JN+NS                                                          SHEAR2   312
      NTOT=2*NSIZT(M)                                                   SHEAR2   313
      PRINT 9003,(CN(NTOT+I),I=1,NANG)                                  SHEAR2   314
 9003 FORMAT(16H TOT PL STRAIN =6F10.3)                                SHEAR2   315
      PRINT 1900,(TAUZ(I),I=1,NANG)                                    SHEAR2   316
 1900 FORMAT (6H TAUZ= 6F10.6)                                          SHEAR2   317
      RETURN                                                           SHEAR2   318
      END                                                              SHEAR2   319
```

FIGURE 42   LISTING OF SUBROUTINE SHEAR2   (Concluded)

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